

## Math 150 Lecture Notes

### The Dot Product

If  $\mathbf{u} = \langle a_1, b_1 \rangle$  and  $\mathbf{v} = \langle a_2, b_2 \rangle$  are vectors, then their **dot product**, denoted by  $\mathbf{u} \cdot \mathbf{v}$ , is defined by

$$\mathbf{u} \cdot \mathbf{v} = a_1 a_2 + b_1 b_2$$

#### Properties of the Dot Product

1.  $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2.  $(a\mathbf{u}) \cdot \mathbf{v} = a(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (a\mathbf{v})$
3.  $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$
4.  $|\mathbf{u}|^2 = \mathbf{u} \cdot \mathbf{u}$

#### The Dot Product Theorem

If  $\theta$  is the angle between two nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$ , then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta \quad \text{and} \quad \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|}$$

Two nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$  are perpendicular or **orthogonal** iff  $\mathbf{u} \cdot \mathbf{v} = 0$ .

Example 1: Find  $\mathbf{u} \cdot \mathbf{v}$  and the angle between  $\mathbf{u}$  and  $\mathbf{v}$  to the nearest degree.

$$\mathbf{u} = \langle -3, 1 \rangle \quad \mathbf{v} = \langle 4, -2 \rangle$$

Example 2: Find  $\mathbf{u} \cdot \mathbf{v}$  and the angle between  $\mathbf{u}$  and  $\mathbf{v}$  to the nearest degree.

$$\mathbf{u} = -2\mathbf{i} + \mathbf{j} \quad \mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$$

Example 3: Determine whether the given vectors are orthogonal.

$$\mathbf{u} = -3\mathbf{i} \quad \mathbf{v} = 4\mathbf{j}$$

Example 4: Find the indicated quantity, assuming  $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ ,  $\mathbf{v} = 3\mathbf{i} + \mathbf{j}$ , and  $\mathbf{w} = -\mathbf{i} + 2\mathbf{j}$

$$\mathbf{v} \cdot (\mathbf{u} + \mathbf{w})$$