Paper Folding Construction

This activity uses paper folding of patty paper (or squares of waxed paper 10-12 cm on a side), compass, ruler, and protractor to construct the  
   a) bisector of an angle,  b) perpendicular bisector of a line segment,  c) incenter of a triangle,  d) circumcenter of a triangle, and  e) centroid of a triangle.

Work in pairs. One student should read the directions and the other should do the folding. Change roles for each construction.

I. Construct the angle bisector.
   1. Draw \( \angle ABC \).
   2. Fold \( BA \) onto \( BC \) and crease the paper.
   3. Open the paper and mark a point \( D \) on the line formed by the crease.
   4. Measure \( \angle ABD \) and \( \angle CBD \). What is true about the measures of \( \angle ABD \) and \( \angle CBD \)? What is the relationship between \( BD \) and \( \angle ABC \)?
   5. Recall that the distance from a point to a line is the length of the perpendicular segment drawn from the point to the line. Place the edge of another piece of paper (or something with a square edge) on side \( BC \) of \( \angle ABC \) so that the adjacent side of the paper passes through point \( D \). Label the point \( E \) on \( BC \) and mark the length \( DE \) on the edge of the second sheet. Then repeat the steps to find a point \( F \) on \( BA \). Compare \( DF \) and \( DE \). What did you find? Place the point of your compass at \( D \), open the compass to \( E \), and draw a circle. What is the relationship between the circle and the sides of \( \angle ABC \)?

II. Construct the perpendicular bisector of a line segment.
   1. Draw a line segment \( PQ \).
   2. Fold \( P \) onto \( Q \) and crease the paper. Open the paper and mark the point \( M \). \( M \) is the \underline{____________________} of \( PQ \).
   3. Mark a point \( R \) on the crease. Measure \( \angle RPM \) and \( \angle RMQ \). What is the measure of each angle?
   4. Describe the relationship between \( RM \) and \( PQ \).
   5. Draw \( RP \) and \( RQ \). Fold the paper on \( RM \) again. What is true about \( RP \) and \( RQ \)?
   6. Choose any other point \( X \) on \( RM \). What is true about \( PX \) and \( QX \)?
   7. What can be said about any point \( X \) on \( RM \) and its relationship to \( P \) and \( Q \)?
III. Construct the incenter of a triangle.

1. Draw $\triangle ABC$ on your paper. Fold the paper to bisect each angle.

2. What appears to be true about the three angle bisectors?

3. Label the point where the bisectors intersect $P$. Place the edge of a second piece of paper (or something with a square edge) along one side of the triangle so that the adjacent side passes through $P$. Label the point $Q$ on $\overline{AB}$.

4. To determine whether $P$ is equidistant from the three sides of the triangle, place the point of your compass at $P$, open the compass to point $Q$, and draw a circle. The point $P$ is called the incenter of the triangle.

IV. Construction the circumcenter of a triangle.

1. Draw an acute $\triangle DEF$.

2. Fold the paper to construct the perpendicular bisector of each side of the triangle. What appears to be true about the perpendicular bisectors?

3. Label the point of intersection $Q$. Measure $DQ$, $EQ$, and $FQ$. What did you find?

4. Place the point of your compass at $Q$, open it to $D$, and draw a circle. $Q$ is called the circumcenter of the triangle.

5. Repeat the steps above for an obtuse triangle and a right triangle. How are the results the same? How are they different?

V. Construct the centroid of a triangle.

1. Draw $\triangle XYZ$. Fold the paper to locate the midpoints of the sides of the triangle and label them $A$, $B$, and $C$.

2. Draw the medians $\overline{XC}$, $\overline{YB}$, and $\overline{ZA}$. What appears to be true about the medians?

3. Label the point of intersection, called the centroid, $M$. Measure the two segments on each median and find the ratio of the length of the shorter segment to the length of the longer segment. The centroid divides each median into two segments so that the ratio of their lengths is ______________.