Basic Postulates & Theorems of Geometry

Postulates

Postulates are statements that are assumed to be true without proof. Postulates serve two purposes - to explain undefined terms, and to serve as a starting point for proving other statements.

Euclid's Postulates

Two points determine a line segment.
A line segment can be extended indefinitely along a line.
A circle can be drawn with a center and any radius.
All right angles are congruent.
If two lines are cut by a transversal, and the interior angles on the same side of the transversal have a total measure of less than 180 degrees, then the lines will intersect on that side of the transversal.

Point-Line-Plane Postulates

Unique Line Assumption: Through any two points, there is exactly one line.
Dimension Assumption: Given a line in a plane, there exists a point in the plane not on that line. Given a plane in space, there exists a line or a point in space not on that plane.
Number Line Assumption: Every line is a set of points that can be put into a one-to-one correspondence with real numbers, with any point on it corresponding to zero and any other point corresponding to one. This was once called the **Ruler Postulate**.
Distance Assumption: On a number line, there is a unique distance between two points.
If two points lie on a plane, the line containing them also lies on the plane.
Through three noncolinear points, there is exactly one plane.
If two different planes have a point in common, then their intersection is a line.

Theorems

Theorems are statements that can be deduced and proved from definitions, postulates, and previously proved theorems.

Line Intersection Theorem: Two different lines intersect in at most one point.
Betweenness Theorem: If C is between A and B and on $\overline{AB}$, then AC + CB = AB.
Related Theorems:

- **Theorem:** If A, B, and C are distinct points and AC + CB = AB, then C lies on $\overline{AB}$.
- **Theorem:** For any points A, B, and C, AC + CB $\geq \overline{AB}$.

Pythagorean Theorem: $a^2 + b^2 = c^2$, if c is the hypotenuse.

Angle Pairs

Complementary angles sum to 90 degrees.

Supplementary angles sum to 180 degrees.

Two angles that are both complementary to a third angle are congruent.

Two angles that are both supplementary to a third angle are congruent.

Vertical angles are congruent.

Special Triangles

The base angles of an isosceles triangle are congruent.

The legs of an isosceles triangle are congruent.

The sides of an equilateral triangle are equal.

The angles of an equilateral triangle are equal.

The acute angles of a right triangle are complementary.

The altitude to the hypotenuse of a right triangles forms two similar triangles that are also similar to the original triangle.

The length of the median to the hypotenuse is 1/2 the length of the hypotenuse.

Triangle Angles and Sides

The sum of the angles of a triangle is 180 degrees.

The measure of an exterior angle of a triangle is equal to the sum of the remote interior angles.

The measure of an exterior angle of a triangle is greater than that of either remote interior angle.

When two angles of a triangle are equal, their opposite sides are equal, and vice versa.

When two angles of a triangle are unequal, their opposite sides are unequal, and vice versa.

When two sides of a triangle are unequal, the longer side is opposite the larger angle, and vice versa.

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Parallel/Perpendicular Lines

There exists one line parallel to a given line through a fixed point.

If two lines are each parallel to a third line, then they are parallel to each other.

When parallel lines are cut by a transversal, alternate interior, alternate exterior, and corresponding angles are congruent.

When parallel lines are cut by a transversal, interior angles on the same side of the transversal are supplementary.

Every perpendicular segment that joins two parallel lines has the same length.

The points along a perpendicular bisector are equidistant from the endpoints of the segment it bisects.

Properties of Polygons

The angle sum of a quadrilateral is 360 degrees.

The angle sum of any n-sided polygon is $180(n - 2)$ degrees.

The number of diagonals of any n-sided polygon is $1/2(n - 3)n$.

The sum of the exterior angles of a polygon is 360 degrees.

The radii of a regular polygon bisect the interior angles.

The central angles of a regular polygon are congruent.

The apothems of a regular polygon are contained in the perpendicular bisectors of each side.

Each apothem of a regular polygon bisects the central angle whose rays intersect the polygon at the vertices of the side to which the apothem is drawn.

Quadrilaterals

Both pairs of opposite sides and opposite angles in a parallelogram are congruent.

The consecutive angles of a parallelogram are supplementary.

The diagonals of a parallelogram bisect each other.

The diagonals of a rhombus are contained in each other's perpendicular bisector.

The diagonals of a rhombus bisect its interior angles.

The diagonals of a rectangle are congruent.

The base angles, legs, and diagonals of an isosceles trapezoid are congruent.

The median of a trapezoid is parallel to its bases and the average of their lengths.

A quadrilateral is a parallelogram if (1) it has one pair of sides that are both parallel and congruent, (2) both pairs of opposite sides are congruent, (3) Both pairs of opposite angles are congruent, or (4) Its diagonals bisect each other.

Segments Within Triangles

The angle bisectors of a triangle intersect at the incircle of that triangle.

The angle bisectors of a triangle divide the opposite side into two segments proportional to the lengths of the other sides.

The perpendicular bisectors of the sides of a triangle intersect at the circumcircle of that triangle.

The altitudes of a triangle intersect at the orthocenter of that triangle.

The medians of a triangle intersect at the centroid of that triangle.

The midsegments of a triangle are parallel to the side with which they don't intersect, and half the length of that side.

A line parallel to one side of a triangle that intersects with the other two sides divides those sides proportionally.

The proportion of the lengths of the altitudes of similar triangles is the same as that between the corresponding sides of those triangles.

The proportion of the lengths of the medians of similar triangles is the same as that between the corresponding sides of those triangles.

Circles

The radii of a circle are congruent.

All diagonals of a circle are congruent.

Segments in Circles

The perpendicular bisector of a chord contains the center of the circle.

A diameter that bisects a chord is perpendicular to it.

A diameter that is perpendicular to a chord bisects it.

When chords intersect in the same circle, the products of their segments are equal.

Parallel chords cut congruent arcs.

Congruent chords in the same circle are equidistant from the center.

Congruent chords in the same circle define (cut) congruent arcs.

Segments Outside Circles

A tangent line is perpendicular to the radius whose endpoint is the point of tangency.

Tangent segments from the same exterior point are congruent.

When two secant segments share the same exterior endpoint, the products of the secant segments and their external segments are equal.

When a tangent segment and a secant segment share an exterior endpoint, the square of the length of the tangent segment is equal to the product of the secant segment with its external segment.

Angles and Circles

The measure of an inscribed angle is half the measure of its intercepted arc.

The measure of an angle whose vertex is on the circle, whose sides are a chord and a tangent segment, is half the measure of the arc it intercepts.

The measure of an angle whose sides are contained in distinct secant lines and whose vertex is in the interior of a circle is equal to half the sum of the measures of its intercepted arcs.

The measure of an angle whose vertex lies outside a circle, whose sides, when extended, both intersect the circle, is equal to half the difference of the measures of its intercepted arcs.

The measure of a central angle is equal to the measure of the arc it intercepts.

Congruence

When the corresponding parts of triangles are all equal, the triangles are congruent.

When triangles are congruent, all of their corresponding parts are equal.