Math 403

Lesson 9—Recursion, Sequences, and Series

MODE

HORIZ (split screen horizontally)
G-T (split screen vertically)—correlates window step value and table set values

TABLE SET

AUTO-AUTO
ASK-AUTO
AUTO-ASK
ASK-ASK

RECURSION in the home screen

Suppose a bacteria cell divides once a minute when we start with one bacterium. Use the home screen of your graphing calculator to model the growth. How many cells will we have after one hour? How many after two hours? How long will it take for one cell to divide so that there one billion cells are formed?

Allergy season has hit your area! You decide to take 20 mg of a popular allergy medication, No-Drip. After 24 hours, approximately 40% of the No-Drip has been flushed from your system.

How much No-Drip is still in your system at the start of day 2?

How much No-Drip is still in your system at the start of day 3?

Graph the amount of No-Drip that is still in your system after *n* days.

How Much No-Drip?

Allergy season has hit your area! You decide to take 20 mg of a popular allergy medication, No-Drip. After 24 hours, approximately 40% of the No-Drip has been flushed from your system.

1. How much No-Drip is still in your system at the start of day 2?

2. How much No-Drip is still in your system at the start of day 3?

3. Graph the amount of No-Drip that is still in your system after *n* days. Use the Sequence mode and write the sequence:

Have you ever wondered why you must take some medications for a few days before they reach maximum effectiveness?

4. You begin to take 20 mg of No-Drip once a day every day. When you take the 20 mg does the second day, how much No-Drip is in your system?

5. How much No-Drip will you have in your system on day three after taking your daily 20 mg dose?

6. Predict: Will the amount of No-Drip in your system continue to rise without bound?

7. Use the sequence mode in your calculator to graph the amount of No-Drip that is in your system after n days.

Consider the maximum amount of No-Drip in your system each day.

Day	1	2	3	4
Amount of No-Drip	20	32	39.2	43.52
		12	7.2	4.32

The sequence above can be seen as a sequence of partial sums, S_{day}.
$S_1 = 20$
$S_2 = 20 + 0.60S_1 = 20 + 12 = 32$
$S_3 = 20 + .060S_2 = 20 + 12 + 7.2 = 39.2$
$S_4 = 20 + 0.60S_3 = 20 + 12 + 7.2 + 4.32 = 43.52$

Consider the series: 20 + 12 + 7.2 + 4.32 + …

8. How does the above No-Drip problem relate to finding the sum of a series?

9. What is the minimum and maximum amount of No-Drip that is in your system at any given time?

10. How big of a dose would you want to take if you need to have a minimum of 60 mg and a maximum of 100 mg in your system?

TEKS

(6.4) **Patterns, relationships, and algebraic thinking.** The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:
(A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area.
(6.5) **Patterns, relationships, and algebraic thinking.** The student uses letters to represent an unknown in an equation. The student is expected to formulate equations from problem situations described by linear relationships.
(7.4) **Patterns, relationships, and algebraic thinking.** The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:
(C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.
(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:
(B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).
(A.3) **Foundations for functions.** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to:
(A) use symbols to represent unknowns and variables; and
(B) look for patterns and represent generalizations algebraically.
(A.7) **Linear functions.** The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
(A) analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;
(2A.11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to:
(D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods;
(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.
(P.4) The student uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems. The student is expected to:
(A) represent patterns using arithmetic and geometric sequences and series;
(B) use arithmetic, geometric, and other sequences and series to solve real-life problems.