Math 131 Final Review
May 2010

1. Find the equation of the tangent line to $y = 3x^2 - 2x + 1$ at the point (1, 2). $m = 4$

$y' = 6x - 2$

$y'(1) = 6(1) - 2 = 4$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$\boxed{y = 4x - 2}$

2. A point on a turning wheel is distance y from the road at time t. Using the data given in the chart below, estimate the speed after 2.3 seconds using the average of two secant lines.

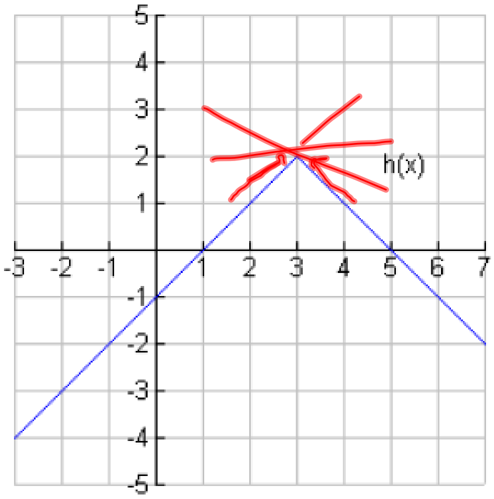
Time, t	Distance, d ft
2.1	1.2
2.2	1.5
2.3	1.7
2.4	2.0
2.5	2.3

$m_1 = \frac{1.7 - 1.5}{2.3 - 2.2} = \frac{.2}{.1} = 2$

$m_2 = \frac{2.0 - 1.7}{2.4 - 2.3} = \frac{.3}{.1} = 3$

$\frac{m_1 + m_2}{2} = \frac{2 + 3}{2} = 2.5$ ft/sec

3. Find (a) $\lim_{x \to 3} h(x)$ and (b) $h'(3)$



a) $\lim_{x \to 3} h(x) = 2$

left + right limits both equal 2

b) $h'(3)$ DNE

undef

4. Find $\lim_{h \to 0} \frac{(x+h)^2}{h} = \lim_{h \to 0} \frac{x^2+2xh+h^2}{h}$ DNE

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5. Find $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25} \cdot \frac{\sqrt{x}+5}{\sqrt{x}+5} = \lim_{x\to 25} \frac{\cancel{x-25}}{\cancel{(x-25)}(\sqrt{x}+5)} = \lim_{x\to 25} \frac{1}{\sqrt{x}+5} = \frac{1}{\sqrt{25}+5} = \boxed{\frac{1}{10}}$

6. Find $\lim_{x\to\infty} \frac{x^2+2x}{2x^2-2x+1} = \lim_{x\to\infty} \frac{\frac{x^2}{2x^2}+\frac{2x}{2x^2}}{\frac{2x^2}{2x^2}-\frac{2x}{2x^2}+\frac{1}{2x^2}} = \lim_{x\to\infty} \frac{\frac{1}{2}+\frac{1}{x}}{1-\frac{1}{x}+\frac{1}{2x^2}} = \boxed{\frac{1}{2}}$

7. Find any and all values where $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 - 5x & \text{if } 0 \le x \le 5 \\ 5 & \text{if } x > 5 \end{cases}$ is discontinuous.

$\lim_{x\to 5^-} f(x) = 0$
$\lim_{x\to 5^+} f(x) = 5$ } not the same
$f(5) = 0$

$\lim_{x\to 0^-} f(x) = 0$
$\lim_{x\to 0^+} f(x) = 0$
$f(0) = 0$

continuous @ $x=0$

$\boxed{\text{discontinuous @ } x=5}$

8. Epidemiologists in College Station, Texas, estimate that t days after the flu begins to spread in town, the percent of the population infected by the flu is approximated by $p(t) = t^2 + t$.

Lial, M. L., Greenwell, R. N., & Ritchey, N. P. (2002). *Finite Mathematics and Calculus with Applications* (6th ed.). Boston: Pearson.

(a) Find the average rate of change of p with respect to t over the interval from 1 to 4 days.

$p(1) = 1^2 + 1 = 2$

$p(4) = 4^2 + 4 = 20$

$\frac{20-2}{4-1} = \frac{18}{3} = 6$ percent/day

(b) Find the instantaneous rate of change of p with respect to t at $t = 3$.

$p'(t) = 2t + 1$

$p'(3) = 2(3) + 1 = 7$ percent/day

9. Lindsay Branson finds that, after introducing her dog Buddy to a new brand of food, Buddy's weight begins to increase. After x weeks on the new food, Buddy's weight (in pounds) is approximately given by $w(x) = \sqrt{x} + 17$, for $0 \le x \le 6$. Find the rate of change of Buddy's weight after x weeks, using the definition of derivative.

Lial, M. L., Greenwell, R. N., & Ritchey, N. P. (2002). *Finite Mathematics and Calculus with Applications* (6th ed.). Boston: Pearson.

$$w'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} + 17 - (\sqrt{x} + 17)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} + \cancel{17} - \sqrt{x} - \cancel{17}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

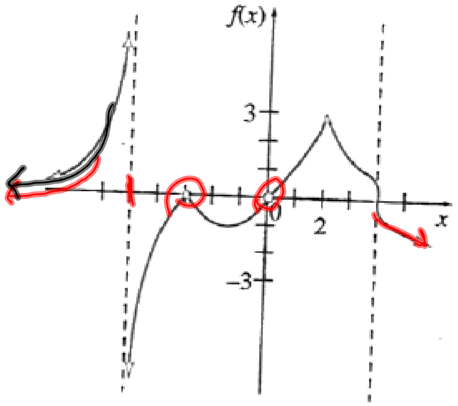
$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$\boxed{w'(x) = \frac{\sqrt{x}}{2x}}$$

10. Give the x-values where the graph shown below is not differentiable.



1) discontinuos $-5, -3, 0$
2) cusp 2
3) vertical tangent line 4

$\boxed{-5, -3, 0, 2, 4}$

11. Find the infinite limits, limits at infinity, and asymptotes for the function shown in #10.

$\lim_{x \to -5^-} f(x) = \infty$

$\lim_{x \to -5^+} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = -\infty$

asymptotes

$x = -5$

$y = 0$

12. In an experiment testing methods of sexually attracting male insects to sterile females, equal numbers of males and females of a certain species are permitted to intermingle. Assume that $M(t) = 4t^{3/2} + 2t^{1/2}$ approximates the number of matings observed among the insects in an hour, where t is the temperature in degrees Celsius. Find the rate of change in the number of matings at 25°C.

Lial, M. L., Greenwell, R. N., & Ritchey, N. P. (2002). *Finite Mathematics and Calculus with Applications* (6th ed.). Boston: Pearson.

$$M'(t) = \frac{3}{2}(4)t^{1/2} + \frac{1}{2}(2)t^{-1/2}$$
$$= 6t^{1/2} + t^{-1/2}$$
$$= 6\sqrt{t} + \frac{1}{\sqrt{t}}$$

$$M'(25) = 6\sqrt{25} + \frac{1}{\sqrt{25}}$$
$$= 30 + \frac{1}{5}$$
$$= \boxed{30.2 \text{ matings/hour}}$$

13. Find the velocity when $t = 2$ for a rock dropped from a 144-foot building, given that its position in feet above the ground is $s(t) = -16t^2 + 144$, where t is the time in seconds since it was dropped.

$$v(t) = s'(t) = -32t$$
$$v(2) = -32(2) = \boxed{-64 \text{ ft/sec}}$$

14. When will the rock in #13 hit the ground? What is its velocity upon impact?

time

$$s(t) = 0$$
$$-16t^2 + 144 = 0$$
$$16t^2 = 144$$
$$t^2 = 9$$
$$t = \pm 3$$

no neg time

$$\boxed{3 \text{ sec}}$$

$$v(3) = -32(3) = \boxed{-96 \text{ ft/sec}}$$

15. Assume the total number (in millions) of bacteria present in a culture at a certain time t (in hours) is given by $N(t) = (t - 10)^2 (2t) + 50$. Find and interpret $N'(8)$.
Adapted from Lial, M. L., Greenwell, R. N., & Ritchey, N. P. (2002). *Finite Mathematics and Calculus with Applications* (6th ed.). Boston: Pearson.

$$N'(t) = (t-10)^2(2) + 2(t-10)(2t) + 0$$
$$= 2(t^2 - 20t + 100) + 4t(t-10)$$
$$= 2t^2 - 40t + 200 + 4t^2 - 40t$$
$$= 6t^2 - 80t + 200$$
$$N'(8) = -56 \text{ million bacteria/hr}$$

After 8 hours, the number of bacteria is decreasing at the rate of 56 million per hour.

16. To test an individual's use of calcium, a researcher injects a small amount of radioactive calcium into the person's bloodstream. The calcium remaining in the bloodstream is measured each day for several days. Suppose the amount of the calcium remaining in the bloodstream (in milligrams per cubic centimeter) t days after the initial injection is approximated by $C(t) = \frac{1}{2}(2t + 1)^{-1/2}$. Find the rate of change of the calcium level with respect to time for 4 days.

$$C'(t) = \frac{1}{2}\left(-\frac{1}{2}\right)(2t+1)^{-3/2}(2)$$
$$= -\frac{1}{2(2t+1)^{3/2}}$$
$$C'(4) = -\frac{1}{2[2\cdot 4+1]^{3/2}} = -\frac{1}{2(9)^{3/2}} = \frac{-1}{2(27)} = \frac{-1}{54}$$
$$C'(4) = -\frac{1}{54} \text{ mg/cc/day}$$

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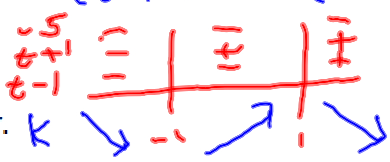
17. Suppose a certain drug is administered to a patient, with the percent of concentration of the drug in the bloodstream t hours later given by $K(t) = \frac{5t}{t^2+1}$. On what time intervals is the concentration of the drug increasing? On what intervals is it decreasing?

K' pos/neg

$$K'(t) = \frac{(t^2+1)5 - 5t(2t)}{(t^2+1)^2} = \frac{5t^2+5-10t^2}{(t^2+1)^2} = \frac{-5t^2+5}{(t^2+1)^2} = \frac{-5(t^2-1)}{(t^2+1)^2}$$

$K'(t) = 0$ when $t = \pm 1$



$\boxed{\begin{array}{l} \text{inc } (0,1) \\ \text{dec } (1,\infty) \end{array}}$

18. Find all local extrema for $g(x) = x^2 + 1/x$.

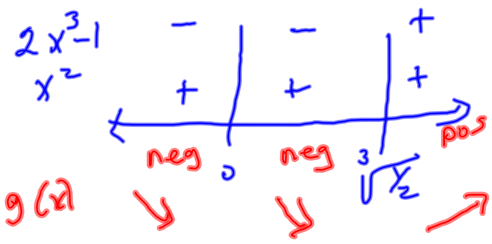
$$g'(x) = 2x - x^{-2} = 2x - \frac{1}{x^2} = \frac{2x^3-1}{x^2}$$

und @ $x=0$

$2x^3 - 1 = 0$

$x^3 = \frac{1}{2}$

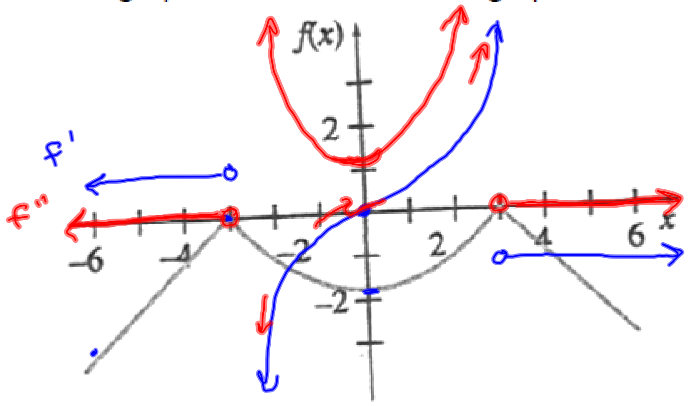
$x = \sqrt[3]{1/2}$



local minimum @ $x = \sqrt[3]{1/2}$

$g(\sqrt[3]{.5}) = (\sqrt[3]{.5})^2 + \frac{1}{\sqrt[3]{.5}} \approx 1.89$ min. value

19. Sketch the graphs of f' and f'' on the graph below.



$(-3,0)$ $(-6,-3)$

$m = \frac{-3-0}{-6--3}$

$= \frac{-3}{-3}$

$= 1$

20. For $h(x) = -2x^3 + 9x^2 + 168x - 3$, find the intervals where the function is concave upward and intervals where it is concave downward. Find any inflection points.

$h'(x) = -6x^2 + 18x + 168$

$h''(x) = -12x + 18$

$= -6(2x-3)$

$0 = -6(2x-3)$

$x = 3/2$

$-12x+18$ + | −

h CU | 3/2 | CD

$\boxed{\begin{array}{ll} CU & (-\infty, 3/2) \\ CD & (3/2, \infty) \end{array}}$

$h(\frac{3}{2}) = -2(\frac{3}{2})^3 + 9(\frac{3}{2})^2 + 168(\frac{3}{2}) - 3$

$\boxed{\text{inflection pt } (1.5, 262.5)}$

21. Find the absolute maxima and minima for $f(x) = \frac{x-1}{x^2+1}$ over the interval [1, 5].

$$f'(x) = \frac{(x^2+1)1-(x-1)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2+2x}{(x^2+1)^2} = \frac{-x^2+2x+1}{(x^2+1)^2}$$

$-x^2+2x+1=0$

$x^2-2x-1=0$

$$x = \frac{2 \pm \sqrt{(-2)^2-4(-1)}}{2} = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$f(1) = 0$ min

$f(5) \approx .154$

$f(1+\sqrt{2}) \approx .207$ max

$f(1-\sqrt{2}) \approx -1.207$ min

neg

not in interval

22. A company wants to manufacture cylindrical aluminum cans with a volume of 1000 cubic centimeters. What should the radius and height of the can be to minimize the amount of aluminum used?

minimize S.A.

$V = \pi r^2 h$

$\pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

$A(r) = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2000}{r}$ $\quad 2000r^{-1}$

$A'(r) = 4\pi r - 2000r^{-2}$

$= 4\pi r - \frac{2000}{r^2}$

$= \frac{4\pi r^3 - 2000}{r^2}$

$4\pi r^3 - 2000 = 0$

$r^3 = \frac{2000}{4\pi}$

$r = \sqrt[3]{\frac{2000}{4\pi}}$

$\boxed{r = \sqrt[3]{\frac{500}{\pi}} \approx 5.419 \text{ cm}}$

$\boxed{h = \frac{1000}{\pi r^2} \approx 10.839 \text{ cm}}$

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23. Use the differential to approximate $\sqrt{145}$.

Since $\sqrt{144}=12$, $x=144$ $\Delta x = dx = 1$

Use dy to approximate $\Delta y = \sqrt{145}-\sqrt{144}$

Since $\sqrt{x} = x^{1/2}$, $f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $dy = \frac{1}{2\sqrt{x}}dx$

Substituting $x = 144$ $dy = \frac{1}{2\sqrt{144}} = \frac{1}{24}$ So $\sqrt{145} = f(x+\Delta x) \approx f(x)+dy$

$= \sqrt{x}+dy$

$= \sqrt{144}+\frac{1}{24}$

$= 12+\frac{1}{24}$

$= \boxed{12\frac{1}{24}}$

24. Find $\int [x^2(x^4+4x+3)]dx$.

$= \int (x^6+4x^3+3x^2)\,dx$

$= \frac{x^7}{7}+\frac{4x^4}{4}+\frac{3x^3}{3}+C$

$= \boxed{\frac{1}{7}x^7+x^4+x^3+C}$

25. Find $\int_{-2}^{3}\frac{-4x}{x^2+3}dx$.

$= -2\int_{-2}^{3}(x^2+3)^{-1}\,2x\,dx$ Let $u = x^2+3$

$du = 2x\,dx$

$= -2\int_{u(-2)}^{u(3)} u^{-1}\,du$

$= -2\ln|u|\Big]_{u(-2)}^{u(3)}$

$= -2\ln|x^2+3|\Big]_{-2}^{3}$

$= -2\ln|9+3|+2\ln|4+3|$

$= -2\ln 12+2\ln 7$

$= 2(-\ln 12+\ln 7)$

$= 2\ln\frac{7}{12}$

$= \ln\frac{49}{144}$

26. Using 4 equal subdivisions and right endpoints, approximate the area under the graph of $f(x) = -x^2 + 4$ from $x = \cancel{-2}$ to $x = \cancel{2}$.
-3 5

$\frac{5--3}{4} = 2$

x	$f(x)$
-3	-5
-1	$3 \leftarrow$
1	$3 \leftarrow$
3	$-5 \leftarrow$
5	$-21 \leftarrow$

$2(3) + 2(3) + 2(-5) + 2(-21)$

$= 6 + 6 - 10 - 42$

$= \boxed{-40}$

27. Evaluate $\int_{\pi}^{2\pi} \sec x \tan x$.

$= \sec x \Big]_{\pi}^{2\pi}$

$= \sec 2\pi - \sec \pi$

$= \frac{1}{\cos 2\pi} - \frac{1}{\cos \pi}$

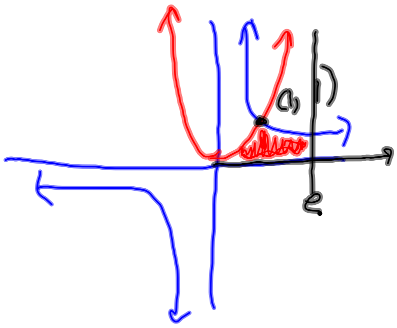
$= \frac{1}{1} - \frac{1}{-1}$

$= 1 + 1$

$= \boxed{2}$

28. Find the area of the region bounded by the curves $y = 1/x$ and $y = x^2$, $y = 0$, and $x = e$.

$$\frac{1}{x} = x^2$$



$$\int_0^1 x^2\,dx + \int_1^e \frac{1}{x}\,dx$$

$$\frac{x^3}{3}\Big]_0^1 + \ln|x|\Big]_1^e$$

$$\frac{1}{3} - 0 + \ln e - \ln 1$$

$$\frac{1}{3} + 1 - 0$$

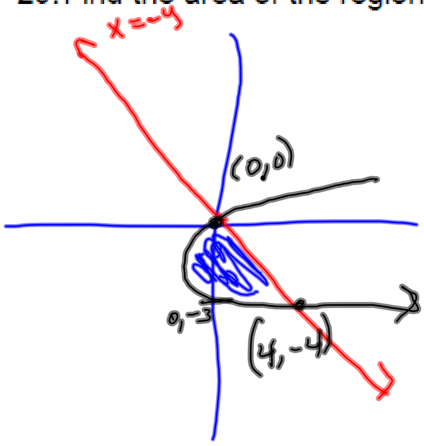
$$\boxed{\frac{4}{3}}$$

29. Find the area of the region bounded by the curves $x + y = 0$ and $x = y^2 + 3y$.

$x = -y$

$y = -x$

$y(y+3)$



$$\Rightarrow -y = y^2 + 3y$$

$$0 = y^2 + 4y$$

$$0 = y(y+4)$$

$$y = 0 \qquad y = -4$$

$$x = -y = 4$$

$$y^2 + 3y = x$$

$$y^2 + 3y - x = 0$$

$$y = \frac{-3 \pm \sqrt{9 - 4(-x)}}{2}$$

$$= \frac{3 \pm \sqrt{9 + 4x}}{2}$$

$$y_1 = \frac{3 + \sqrt{9 + 4x}}{2}$$

$$y_2 = \frac{3 - \sqrt{9 + 4x}}{2}$$

$$\int_{-4}^{0} \left[-y - (y^2 + 3y)\right] dy$$

$$= \int_{-4}^{0} \left(-y - y^2 - 3y\right) dy$$

$$= \int_{-4}^{0} \left(-y^2 - 4y\right) dy$$

$$= -\frac{y^3}{3} - \frac{4y^2}{2}\Big]_{-4}^{0}$$

$$= \frac{(-4)^3}{3} + \frac{4(-4)^2}{2}$$

$$= -\frac{64}{3} + \frac{64}{2}$$

$$= -\frac{64}{3} + 32$$

$$= -\frac{64}{3} + \frac{96}{3}$$

$$= \boxed{\frac{32}{3}}$$

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30. Find the average value of the function $f(x) = x^2\sqrt{1+x^3}$ on the interval [0, 2].

$$\frac{1}{2-0}\int_0^2 \left(x^2\sqrt{1+x^3}\right)dx \qquad \text{Let } u = 1+x^3$$

$$du = \underline{3x^2\,dx}$$

$$= \frac{1}{2}\left(\frac{1}{3}\right)\int_0^2 \sqrt{1+x^3}\;\underline{3x^2\,dx}$$

$$= \frac{1}{6}\int_{u(0)}^{u(2)} u^{1/2}\,du$$

$$= \frac{1}{6}\,\frac{u^{3/2}}{3/2}\Bigg]_{u(0)}^{u(2)}$$

$$= \frac{1}{9}\,u^{3/2}\Bigg]_{u(0)}^{u(2)}$$

$$= \frac{1}{9}(1+x^3)^{3/2}\Bigg]_0^2$$

$$= \frac{1}{9}(1+8)^{3/2} - \frac{1}{9}(1+0)^{3/2}$$

$$= \frac{1}{9}(27) - \frac{1}{9}(1)$$

$$= 3 - \frac{1}{9}$$

$$= 2\,\tfrac{8}{9} \text{ or } \boxed{\frac{26}{9}}$$

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31. Solve the differential equation $\frac{dy}{dx} = 3x^2 \cos(x^3)$

$$dy = 3x^2 \cos(x^3)\, dx$$

$$\int 1\, dy = \int 3x^2 \cos(x^3)\, dx$$

Let $u = x^3$

$du = 3x^2 dx$

$$y = \int \cos u \, du$$

$$y = \sin u + C$$

$$y = \sin(x^3) + C$$

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32. Solve the initial value problem: $\frac{dy}{dt} = 7t^6 - 3t^2 + 5$; $y = 1$ when $t = 1$.

$$dy = (7t^6 - 3t^2 + 5)\,dt$$

$$\int 1\,dy = \int (7t^6 - 3t^2 + 5)\,dt$$

$$y = \frac{7t^7}{7} - \frac{3t^3}{3} + 5t + C$$

$$y = t^7 - t^3 + 5t + C$$

$$1 = 1^7 - 1^3 + 5(1) + C$$

$$1 = 1 - 1 + 5 + C$$

$$C = -4$$

$$\boxed{y = t^7 - t^3 + 5t - 4}$$