Math 131 Week in Review

Sections 4.2-4.3, 4.6, 4.8, 5.1-5.5

4/18/10

f' is 0 or undefined

f(x) graphically: local max/min, $f'(x)=0$

cusp, discontinuity, vertical tangent, $f'(x)$ und

1. Find the critical numbers of $f(x) = \dfrac{x^2 - 5x + 6}{x - 1}$.

$f'(x) = \dfrac{(x-1)(2x-5) - (x^2-5x+6)(1)}{(x-1)^2} = \dfrac{2x^2-7x+5-x^2+5x-6}{(x-1)^2} = \dfrac{x^2-2x-1}{(x-1)^2}$

$x^2-2x-1=0$

$x = \dfrac{2 \pm \sqrt{4-4(-1)}}{2} = \dfrac{2\pm\sqrt{8}}{2} = \dfrac{2\pm 2\sqrt{2}}{2} = \dfrac{\cancel{2}(1\pm\sqrt{2})}{\cancel{2}} = \boxed{1\pm\sqrt{2}}$

und $x-1=0$, $\boxed{x=1}$

2. Find the local extreme values of $g(x) = \begin{cases} 3x + 2x^2 & x < 0 \\ 3 + 2x^2 - x^4 & x \geq 0 \end{cases}$.

$g'(x) = \begin{cases} 3+4x, & x<0 \\ 4x-4x^3, & x\geq 0 \end{cases}$

$4x+3=0$

$x=-3/4$

$4x-4x^3=0$

$4x(1-x^2)=0$

$4x(1+x)(1-x)=0$

$x=0$ $\cancel{x=-1}$ $x=1$

$3+4x$, $4x-4x^3$: − | + | + | − at $-3/4$, 0, 1

f: ↘ $-3/4$ ↗ 0 ↗ 1 ↘

local min (at $-3/4$), local max (at 1)

$g(-\tfrac{3}{4}) = 3(-\tfrac{3}{4}) + 2(-\tfrac{3}{4})^2$

$= -\tfrac{9}{4} + 2(\tfrac{9}{16})$

$= -\tfrac{9}{4} + \tfrac{9}{8}$

$= -\tfrac{18}{8} + \tfrac{9}{8}$

$= \boxed{-\tfrac{9}{8} \text{ local min}}$

$g(1) = 3 + 2(1)^2 - (1)^4$

$= 3+2-1$

$= \boxed{4 \text{ local max}}$

3. Find the global extreme values of $h(x) = x^{\frac{2}{3}}$ over the interval [-2, 3].

$h'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

cannot be 0

und @ $x=0$

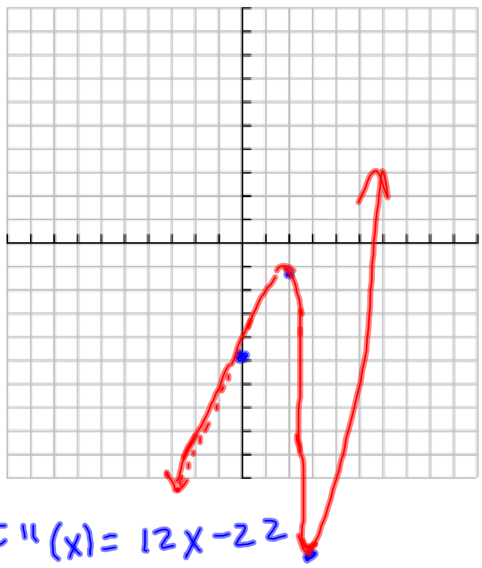
$h(-2) = (-2)^{2/3} = \sqrt[3]{4}$

$h(0) = 0^{2/3} = 0$ min

$h(3) = 3^{2/3} = \sqrt[3]{9}$ max

4. Sketch the graph of $f(x) = 2x^3 - 11x^2 + 12x - 5$ by algebra and calculus methods.

$f(0) = -5$



$f'(x) = 6x^2 - 22x + 12$

$6x^2 - 22x + 12 = 0$

$3x^2 - 11x + 6 = 0$

$(3x-2)(x-3) = 0$

$x = 2/3 \quad x = 3$

$3x-2$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
	$\nearrow$	$2/3$ $\searrow$	3 $\nearrow$

$f\left(\frac{2}{3}\right) = 2\left(\frac{2}{3}\right)^3 - 11\left(\frac{2}{3}\right)^2 + 12\left(\frac{2}{3}\right) - 5$ local max

≈ -1.3

$f(3) = 2(3)^3 - 11(3)^2 + 12(3) - 5$

$= -14$ local min

$f''(x) = 12x - 22$

$12x - 22 = 0$

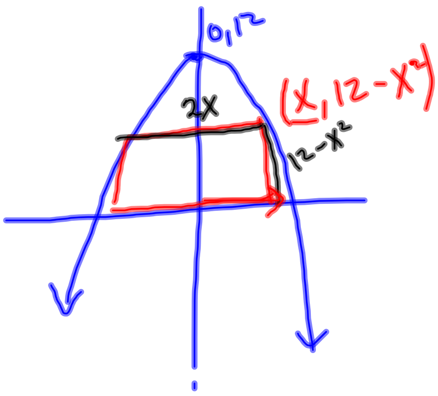
$6x - 11 = 0$

$x = 11/6$

f''	$-$	$+$
	CD $\quad$ 11/6	CU

5. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

max



$A(x) = 2x(12-x^2)$
$= 24x - 2x^3$
$A'(x) = 24 - 6x^2$

$24 - 6x^2 = 0$
$6x^2 = 24$
$x^2 = 4$
$x = \pm 2$
$x = 2$

width $= 2x = 2(2) = 4$
length $= 12 - x^2 = 12 - (2)^2 = 8$

Area $= 4 \times 8 = 32$ sq units

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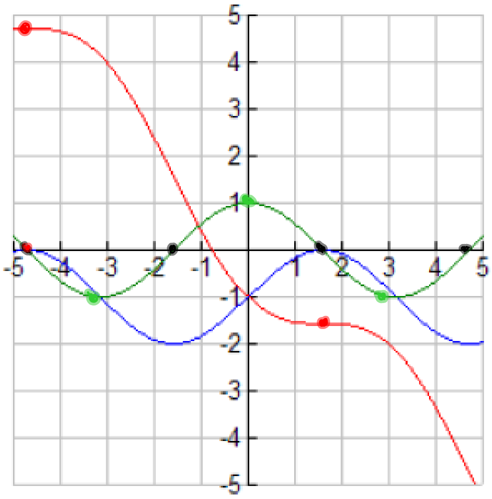
6. The graphs of f, f', and f'' are shown below. Label them appropriately.



green

max/min/slope=0

at $\approx -3, 0, 3$

No values of 0 for red, green

So green $= f''$?

Where does green = 0

blue has local max/min where green = 0

$\therefore$ blue $= f'$

red $= f$

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7. Find the general antiderivative of $g(x) = 3x^2 + 6x - 5 + \cos(2x)$.

$$G(x) = \frac{3x^3}{3} + \frac{6x^2}{2} - 5x + \sin(2x)\left(\frac{1}{2}\right) + C$$
$$= x^3 + 3x^2 - 5x + \frac{1}{2}\sin(2x) + C$$

8. Find the net displacement from 3 to 6 seconds, given the velocity is $v(t) = 4t - 7$ meters per second.

$$\int_3^6 4t-7\,dt$$
$$= \left.\frac{4t^2}{2} - 7t\right]_3^6$$
$$= \left.2t^2-7t\right]_3^6$$
$$= 2(6)^2-7(6)-\left[2(3)^2-7(3)\right]$$
$$= 72-42-[18-21]$$
$$= 30-(-3)$$
$$= 33 \text{ meters}$$

9. Use left endpoints and 5 subintervals to estimate the area under the curve of $y = 3x^2 - 4$ over the interval [-2, 3].

$\frac{3--2}{5} = \frac{5}{5} = 1$

x	y
-2	$3(-2)^2-4=8$
-1	$3(-1)^2-4=-1$
0	$3(0)^2-4)=-4$
1	$3(1)^2-4)=-1$
2	$3(2)^2-4=8$
3	$3(3)^2-4=23$

$L_5 = 1(8)+1(-1)+1(-4)+1(-1)+1(8)$

$= 8-1-4-1+8$

$= 10$

midpts x
-1.5
-.5
.5
1.5
2.5

10. Find the displacement and the distance traveled of a particle over the interval [1, 4], given $v(t) = t - 2$.

$$\int_1^4 (t-2)\,dt = \frac{t^2}{2} - 2t\Big]_1^4 = \frac{4^2}{2} - 2(4) - \left[\frac{1}{2} - 2\cdot 1\right]$$

$$= 8 - 8 - \frac{1}{2} + 2$$

$$= \frac{3}{2}$$

$t-2=0$
$t=2$

neg [1,2) pos (2,4]

$$-\int_1^2 (t-2)\,dt + \int_2^4 (t-2)\,dt$$

$$= -\frac{t^2}{2} + 2t\Big]_1^2 + \frac{t^2}{2} - 2t\Big]_2^4$$

$$= -\frac{4}{2} + 4 - \left[-\frac{1}{2} + 2\right] + \frac{16}{2} - 8 - \left[\frac{4}{2} - 4\right]$$

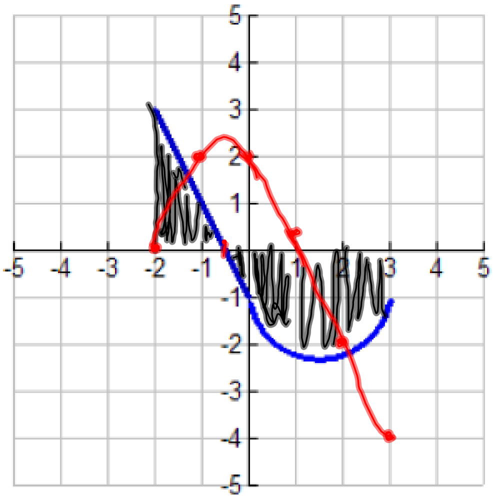
$$= \cancel{-2} + \cancel{4} + \frac{1}{2} - \cancel{2} + \cancel{8} - \cancel{8} - 2 + 4$$

$$= \frac{1}{2} - 2 + 4$$

$$= 2 + \frac{1}{2}$$

$$= \frac{5}{2}$$

11. Let $g(x) = \int_0^x f(t)\,dt$ where f is the function whose graph is shown.



a) Estimate $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$, and $g(3)$.

$g(-2) = 0$

$g(-1) \approx 2$ max

$g(0) \approx 2 + 0 = 2$ max

$g(1) = 2 - 1\frac{3}{4} = \frac{1}{4}$

$g(2) = \frac{1}{4} - 2\frac{1}{4} = -2$

$g(3) \approx -2 - 2 = -4$ min

b) Where does g have a maximum value? Where does it have a minimum value?

max @ $x = -1, 0$ (really about $-.5$)

min @ $x = 3$

c) Sketch the graph of g.

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12. Evaluate:

a) $\int (5e^{2x}-3)\,dx$

Let $u = e^{2x}$

$du = 2e^{2x}\,dx$

$\int 5e^{2x}\,dx - \int 3\,dx$

$= \frac{5}{2}\int 2e^{2x}\,dx - \int 3\,dx$

$= \frac{5}{2}\int 1\,du - \int 3\,dx$

$= \frac{5}{2}u - \int 3\,dx$

$= \frac{5}{2}e^{2x} - 3x + C$

b) $\int \frac{3x^2}{x^3+3}\,dx$

Let $u = x^3+3$

$du = 3x^2$

$= \int u^{-1}\,du$

$= \ln u + C$

$= \ln(x^3+3) + C$

c) $\int x\cos(2x^2)\,dx$

x^2

Let $u = 2x^2$

$du = 4x\,du$

$= \frac{1}{4}\int 4x\cos(2x^2)\,dx$

$= \frac{1}{4}\int \cos u\,du$

$= \frac{1}{4}\sin u + C$

$= \frac{1}{4}\sin 2x^2 + C$

d) $\int \frac{e^x\,dx}{3+e^x}$

Let $u = e^x$

$du = e^x\,dx$

$= \int (3+e^x)^{-1} e^x\,dx$

$= \int (3+u)^{-1}\,du$

$= \ln(3+u) + C$

$= \ln(3+e^x) + C$