Math 131 Week in Review

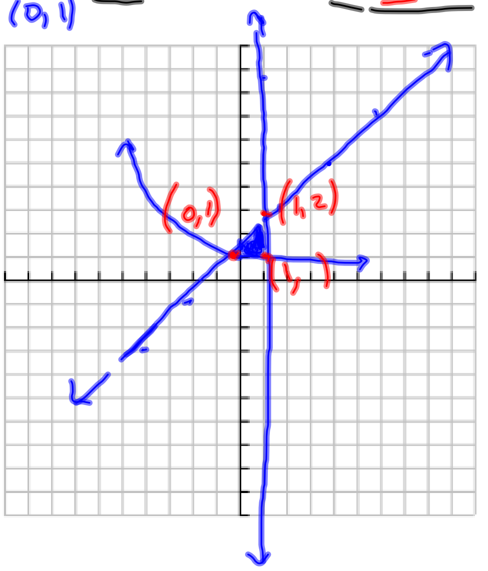
Sections 6.1, 6.5, 6.7

4/25/2010

1. Find the area of the region bounded by the given curves.

   $y = e^{-2x}$ $\qquad$ $y = 1 + x$ $\qquad$ $x = 1$

(0, 1)



$$\int_0^1 \left[(1+x) - e^{-2x}\right] dx$$

$$= x + \frac{x^2}{2} + \frac{1}{2}e^{-2x} \Big]_0^1$$

$$= 1 + \frac{1}{2} + \frac{1}{2}e^{-2} - 0 - 0 - \frac{1}{2}e^0$$

$$= 1 + \cancel{\frac{1}{2}} + \frac{1}{2}e^{-2} - \cancel{\frac{1}{2}}$$

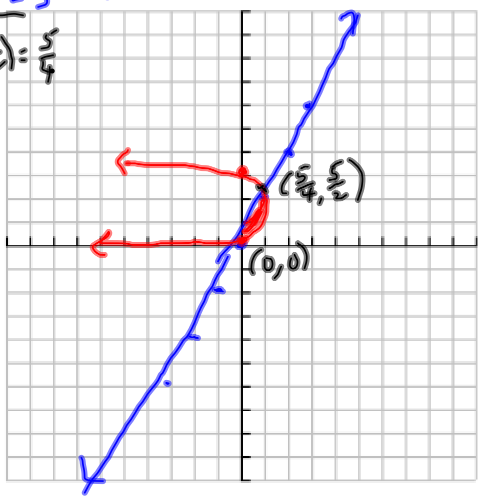
$$= 1 + \frac{1}{2}e^{-2}$$

2. Find the area of the region bounded by the given curves.

$2x - y = 0 \qquad \underline{x = 3y - y^2}$

$\underline{x = \frac{1}{2}y} \qquad y = 2x \qquad x = y(3-y)$

$x = \frac{1}{2}\left(\frac{5}{2}\right) = \frac{5}{4}$



$\frac{1}{2}y = 3y - y^2$

$0 = \frac{5}{2}y - y^2$

$0 = y\left(\frac{5}{2} - y\right)$

$$\int_0^{5/2} \left[(3y - y^2) - \frac{1}{2}y\right] dy$$

$$= \int_0^{5/2} \left(3y - y^2 - \frac{1}{2}y\right) dy$$

$$= \int_0^{5/2} \left(\frac{5}{2}y - y^2\right) dy$$

$$= \left.\frac{5}{2}\frac{y^2}{2} - \frac{y^3}{3}\right]_0^{5/2}$$

$$= \left.\frac{5}{4}y^2 - \frac{1}{3}y^3\right]_0^{5/2}$$

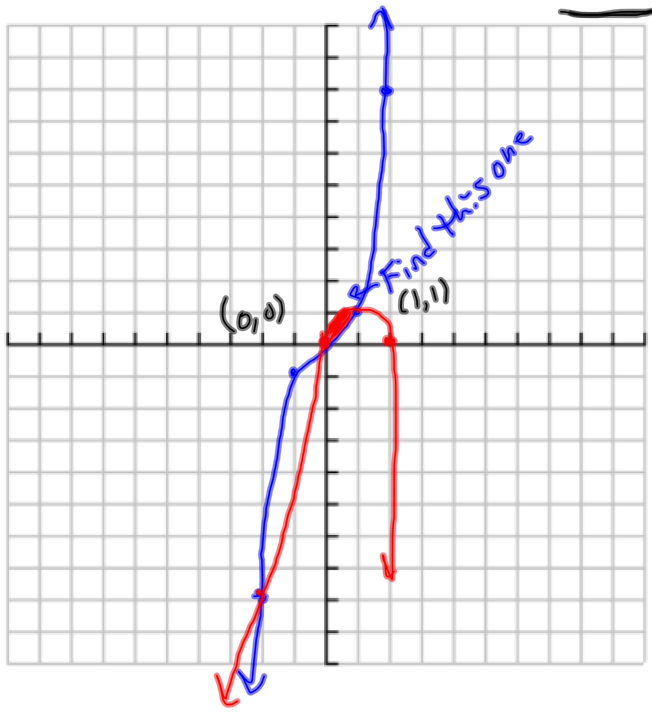
$$= \frac{5}{4}\left(\frac{5}{2}\right)^2 - \frac{1}{3}\left(\frac{5}{2}\right)^3 - 0$$

$$= \frac{125}{48}$$

3. Find the area of the region bounded by the curves $y = x^3$ and $y = 2x - x^2$. down

and the x-axis.

$x(2-x)$



$x^3 = 2x - x^2$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x+2)(x-1) = 0$

$0, -2, 1$

$\int_0^1 [(2x - x^2) - x^3]\,dx$

$= \int_0^1 (2x - x^2 - x^3)\,dx$

$= \frac{2x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \Big]_0^1$

$= 1 - \frac{1}{3} - \frac{1}{4}$

$= \frac{5}{12}$

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4. Find the average value of the function $f(x) = 3x^2\sqrt{1-x^3}$ on the interval [-2, 1].

$$\frac{1}{1--2}\int_{-2}^{1} 3x^2\sqrt{1-x^3}\,dx$$

Let $u = 1-x^3$

$du = -3x^2\,dx$

$$= -\frac{1}{3}\int_{-2}^{1}(1-x^3)^{1/2}(-3x^2)\,dx$$

$$= -\frac{1}{3}\int_{u(-2)}^{u(1)} u^{1/2}\,du$$

$$= -\frac{1}{3}\left.\frac{u^{3/2}}{3/2}\right]_{u(-2)}^{u(1)}$$

$$= -\frac{2}{9}\left.(1-x^3)^{3/2}\right]_{-2}^{1}$$

$$= -\frac{2}{9}(1-1^3)^{3/2} - \left(-\frac{2}{9}(1-(-2)^3)^{3/2}\right)$$

$$= -\frac{2}{9}(0) + \frac{2}{9}(9)^{3/2}$$

$$= \frac{2}{9}(3)^3$$

$$= \frac{2}{9}(27)$$

$$= 6$$

5. Find the average value of the function $g(x) = \text{sin}\,(2x)$ over the interval $[0, \pi/2]$.

$$\frac{1}{2}\cdot\frac{1}{\pi/2-0}\int_0^{\pi/2} 2\sin(2x)\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{\pi}{2}}\left(-\cos(2x)\right)\Big]_0^{\pi/2}$$

$$= -\frac{1}{\pi}\cos\left(2\cdot\frac{\pi}{2}\right) + \frac{1}{\pi}\cos(0)$$

$$= -\frac{1}{\pi}\cos\pi + \frac{1}{\pi}\cos 0$$

$$= -\frac{1}{\pi}(-1) + \frac{1}{\pi}(1)$$

$$= \frac{1}{\pi} + \frac{1}{\pi}$$

$$= \frac{2}{\pi}$$

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6. Find the number(s) a such that the average value of $f(x) = 3 - x^2$ on the interval $[0, a]$ is equal to 0.

$$\frac{1}{a-0} \int_0^a (3 - x^2)\,dx$$

$$= \frac{1}{a}\left(3x - \frac{x^3}{3}\right)\Big]_0^a$$

$$= \frac{1}{a}\left(3a - \frac{a^3}{3}\right)$$

$$= 3 - \frac{1}{3}a^2$$

$$3 - \frac{1}{3}a^2 = 0$$

$$-\frac{1}{3}a^2 = -3$$

$$a^2 = 9$$

$$\boxed{a = \pm 3}$$

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7. If a cup of hot chocolate has temperature 90° C in a room where the temperature is 25° C, then according to Newton's Law of Cooling, the temperature of the hot chocolate after t minutes is $T(t) = 25 + 65e^{-t/50}$. What is the average temperature of the hot chocolate the first hour? $[0, 60]$

$$= (-50)\ \frac{1}{60-0}\int_0^{60}\left(25 + 65e^{-t/50}\right)dt$$

$$= -\frac{50}{60}\left(-\frac{1}{2}x + 65e^{-t/50}\right)\Big]_0^{60}$$

$$= \frac{5}{12}x - \frac{325}{6}e^{-t/50}\Big]_0^{60}$$

$$= \frac{5}{12}(60) - \frac{325}{6}e^{-60/50} - \frac{5}{12}(0) + \frac{325}{6}e^0$$

$$= 25 - \frac{325}{6}e^{-1.2} + \frac{325}{6}$$

$$= \frac{475}{6} - \frac{325}{6}e^{-1.2}$$

$\frac{5}{6}(65)$

$\frac{325}{6}$

325
150
475

8. Use Poiseulle's Law to calculate the rate of flow in a small human artery when η = .026, R = .0085, l = 2 cm, and P = 3800 dynes/cm^2. 3 significant digits

$$F = \frac{\pi P R^4}{8\eta l}$$ derived from calculus

$$F = \frac{\pi(3800)(.0085)^4}{8(.026)(2)} \quad cm^3/sec \qquad cc/sec$$

$$\approx .000150 \quad cc/sec$$

9. The marginal cost function $C'(x)$ was defined to be the derivative of the cost function. The marginal cost of manufacturing x feet of pipe is $C'(x) = 3 - .002x + .0009x^2$ dollars per foot, and the fixed start-up cost is $C(0) = \$40,000$. Use the Net Change Theorem to find the cost of producing the first 1000 feet.

$$C(1000) - C(0) = \int_0^{1000} (3 - .002x + .0009x^2)\,dx$$

$$C(1000) - 40,000 = \int_0^{1000} (3 - .002x + .0009x^2)\,dx$$

$$C(1000) = 40,000 + \left[3x - \frac{.002x^2}{2} + \frac{.0009x^3}{3} \right]_0^{1000}$$

$$= 40,000 + 3000 - 1000 + 300,000$$
$$= \$342,000$$

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10. The marginal revenue from the sale of x units of a product is $R'(x) = 32 - .003x$. If the revenue from the sale of the first 1000 units is \$12,500, find the revenue from the sale of the first 8,000 units.

$$R(8000) - R(1000) = \int_{1000}^{8000} (32 - .003x)\,dx$$

$$R(8000) - 12500 = \left(32x - \frac{.003x^2}{2}\right)\Big]_{1000}^{8000}$$

$$R(8000) = 12500 + 32(8000) - \frac{.003(8000)^2}{2} - 32(1000) + \frac{.003(1000)^2}{2}$$

$$= \$142,000$$