Math 131 Week in Review

Sections 1.5-1.6, 2.1

0/31/10

1. Use the laws of exponents to simplify each of the following:

(a) $\dfrac{(6x^2y^5)^3}{\sqrt[3]{x}} = \dfrac{6^3x^6y^{15}}{x^{1/3}}$

$= 216\,x^{17/3}y^{15}$

(b) $(a^{n-1}b^2)^{n+1} = a^{(n-1)(n+1)}\,b^{2(n+1)}$

$= a^{n^2-1}\,b^{2n+2}$

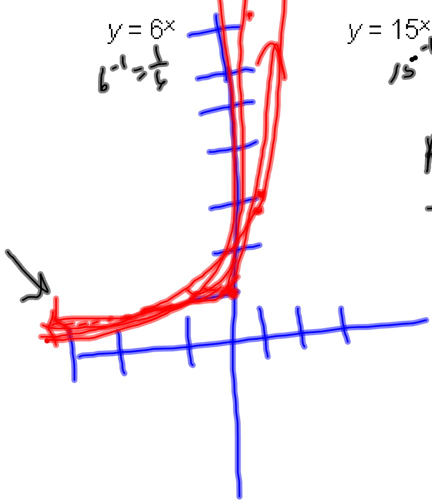
2. Explain the relationship between the graphs of the functions below: $e \approx 2.8\ldots$

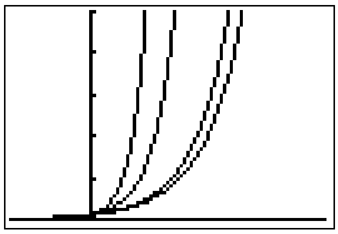
$y = 3^x$ $\quad$ $y = 6^x$ $\quad$ $y = 15^x$ $\quad$ $y = e^x$

$3^{-1} = \frac{1}{3}$ $\quad$ $6^{-1} = \frac{1}{6}$ $\quad$ $15^{-1} = \frac{1}{15}$

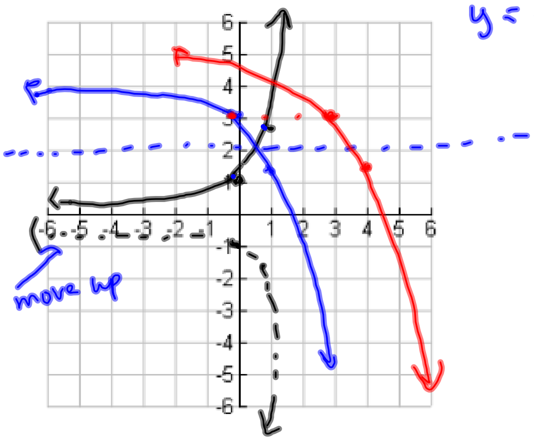
all go through $(0,1)$ and $(1,b)$ where b is the base

As base gets larger, the graph rises faster





3. Starting with the graph of $f(x) = e^x$, write the function g, resulting from reflecting the graph about the line $y = 2$ and translating right 3 units. Sketch the graphs of f and g.



$y = -e^x + 4$

$g(x) = -e^{x-3} + 4$

4. Find the domain of each function.

a. $f(x) = \dfrac{1}{\sqrt{1-2^x}}$

$1-2^x \neq 0$ den.

$1-2^x \geq 0$ radical

$1-2^x > 0$

$-2^x > -1$

$2^x < 1$

$\ln 2^x < \ln 1$

$x \ln 2 < \ln 1$

$x < \dfrac{\ln 1}{\ln 2}$

$\left(-\infty, \dfrac{\ln 1}{\ln 2}\right)$

b. $g(x) = \dfrac{1+2x}{1-e^{1-x^2}}$

$1-e^{1-x^2} \neq 0$

$-e^{1-x^2} \neq -1$

$e^{1-x^2} \neq 1$

$\ln e^{1-x^2} \neq \ln 1$

$1-x^2 \neq \ln 1$

$1-x^2 \neq 0$

$(1+x)(1-x) \neq 0$

$e^{?} = 1$

$e^{0} = 1$

$x \neq -1, 1$

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

5. The half-life of sodium-24, ^{24}Na, is 15 hours.

(a) If a sample has a mass of 30 mg, find the amount remaining after 45 hours.

0	30 mg
15	15 mg
30	7.5 mg
45	3.75 mg

(b) Find the amount remaining after t hours.

$A(t) = 30\left(\frac{1}{2}\right)^{t/15}$

(c) Estimate the amount remaining after 2 days.

$A(48) = 30\left(\frac{1}{2}\right)^{48/15}$

≈ 3.625 mg

(d) Estimate the time required for the mass to be reduced to 2 mg.

$2 = 30\left(\frac{1}{2}\right)^{t/15}$

$\frac{1}{15} = \left(\frac{1}{2}\right)^{t/15}$

$\ln \frac{1}{15} = \ln (.5)^{t/15}$

$\ln \frac{1}{15} = \frac{t}{15} \ln (.5)$

$15 \ln \frac{1}{15} = t \ln (.5)$

$t = \dfrac{15 \ln 1/15}{\ln (.5)}$

$t \approx 58.6$ hours

6. Determine whether the following functions are one-to-one.

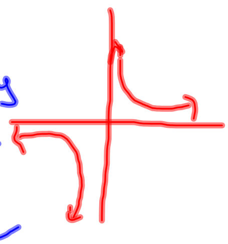
(a) $f(x) = x^2 + 3x - 2$

No - 2 x-values for 1 y

Doesn't pass horizontal line test

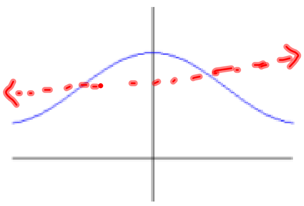
(b) $g(x) = 1/x$ ✓

Yes - 1 x for 1 y
1 y for 1 x



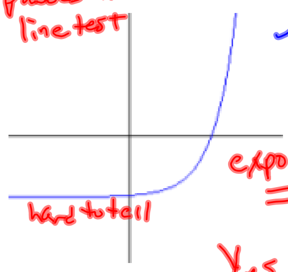
passes horizontal line test

(c)



No, doesn't pass horizontal line test

(d) ✓



hard to tell

exponential

Yes, it passes the horizontal line test

Interchange x and y, and solve for y.

7. Find the inverse of $g(x) = \dfrac{2x+1}{3x-4}$.

$$x = \frac{2y+1}{3y-4}$$

$$x(3y-4) = 2y+1$$

$$3xy-4x = 2y+1$$

$$3xy-2y = 1+4x$$

$$y(3x-2) = 1+4x$$

$$y = \frac{1+4x}{3x-2}$$

$$g^{-1}(x) = \frac{1+4x}{3x-2}$$

8. Find the inverse of $h(x) = \ln(x-2)+1$.

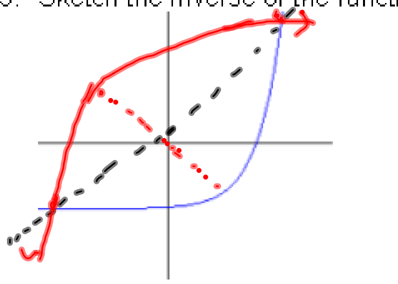
$$x = \ln(y-2)+1$$

$$x-1 = \ln(y-2)$$

$$e^{x-1} = y-2$$

$$y = e^{x-1}+2$$

9. Sketch the inverse of the function graphed below.



To place the line y = x properly, we must assume the x- and y-scale are the same.

Note: the inverse is not a function. It is a relation.

10. If $f(x) = x^2 - 3x + 1$, find $f^{-1}(1)$ and $f^{-1}(-1)$.

$f(1)$ the x-value is 1

$f^{-1}(1)$ the y-value is 1 (original function)

$1 = x^2 - 3x + 1$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0$ or $x = 3$

$f^{-1}(1) = 0$ or $f^{-1}(1) = 3$

$-1 = x^2 - 3x + 1$

$0 = x^2 - 3x + 2$

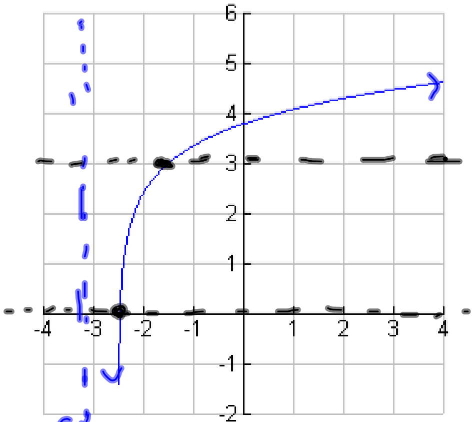
$0 = (x-2)(x-1)$

$x - 2 = 0$ or $x - 1 = 0$

$x = 2$ or $x = 1$

$f^{-1}(-1) = 2$ or $f^{-1}(-1) = 1$

11. The graph of f is given.



f:

D: $(-3, \infty)$

R: $(-\infty, \infty)$

a. What are the domain and range of f^{-1}?

D: $(-\infty, \infty)$

R: $(-3, \infty)$

b. Estimate the value of $f^{-1}(3)$.

looking for y-value of f^{-1}

same as x-value of f

what is x when $y = 3$ for f.

$f^{-1}(3) \approx -1.5$

c. Estimate the value of $f^{-1}(0)$.

what is x when $y = 0$ for f

$f^{-1}(0) \approx -2.5$

12. Find the inverse of $\overset{y}{f}(x) = \dfrac{e^x}{1-2e^x}$.

$x = \dfrac{e^y}{1-2e^y}$

$x(1-2e^y) = e^y$

$x - 2xe^y = e^y$

$e^y + 2xe^y = x$

$e^y(1+2x) = x$

$e^y = \dfrac{x}{1+2x}$

$\ln e^y = \ln \dfrac{x}{1+2x}$

$y = \ln \dfrac{x}{1+2x}$

$f^{-1}(x) = \ln \dfrac{x}{1+2x}$

13. Find the exact value of each of the following.

(a) $2\log_6 36$

$= 2\log_6 6^2$

$= 2(2)$

$= 4$

(b) $\ln \dfrac{1}{e}$

$= \ln e^{-1}$

$= -1$

(c) $\log \sqrt[3]{10}$

$= \log_{10} 10^{1/3}$

$= 1/3$

14. Express as a single logarithm: $\log_2 \sin x \overset{1}{+} \log_2 (x+3) \overset{1}{-} \underset{2}{½} \log_2 5$

$\log_2 [(x+3)\sin x] - \frac{1}{2}\log_2 5$ Law 1

$= \log_2 [(x+3)\sin x] - \log_2 5^{1/2}$ Law 3

$= \log_2 [(x+3)\sin x] - \log_2 \sqrt{5}$

$= \log_2 \dfrac{(x+3)\sin x}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$ Law 2

$= \log_2 \dfrac{\sqrt{5}(x+3)\sin x}{5}$ simplest radical form

1) $\log x + \log y = \log xy$

2) $\log x - \log y = \log \dfrac{x}{y}$

3) $\log x^n = n \log x$

15. Solve for x: $\ln x - \ln(x+2) = 1$

$\boxed{x>0}$ $\quad x+2>0$, $x>-2$

$\ln \frac{x}{x+2} = 1$

$e^1 = \frac{x}{x+2}$

$e(x+2) = x$

$xe + 2e = x$

$xe - x = -2e$

$x(e-1) = -2e$

$x = \frac{-2e}{e-1}$

Since the value is <0, there is no solution.

16. Solve for x: $e^{2x} + 3e^x = 10$

$e^{2x} + 3e^x - 10 = 0$

Let $e^x = y$

$y^2 + 3y - 10 = 0$

$(y+5)(y-2) = 0$

$y+5=0$ or $y-2=0$

$y=-5$ or $y=2$

$\cancel{e^x = -5}$ or $e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

17. Solve for x: $e^{2x-3} > 5$

$\ln e^{2x-3} > \ln 5$

$2x-3 > \ln 5$

$2x > \ln 5 + 3$

$x > \frac{1}{2}\ln 5 + \frac{3}{2}$

or

$x > \ln\sqrt{5} + 1.5$

Jan 28-11:42 AM

18. The table shows the position of a walker.

t (seconds)	0	1	2	3	4	5
d (yards)	0	2.5	6.5	9.5	12	15.5

rate of change for distance over time

Find the average velocity for each time period:

Same as average rate of change

(a) [1, 3]

$(1, 2.5)$ $(3, 9.5)$

$V_{avg} = \frac{9.5-2.5}{3-1}$

$= \frac{7}{2}$

$= 3.5$ yards/sec

(b) [2, 3]

$(2, 6.5)$ $(3, 9.5)$

$V_{avg} = \frac{9.5-6.5}{3-2}$

$= 3$ yards/sec

(c) [3, 4]

$(3, 9.5)$ $(4, 12)$

$V_{avg} = \frac{12-9.5}{4-3}$

$= 2.5$ yards/sec

A(q) - linear

constant rate of change