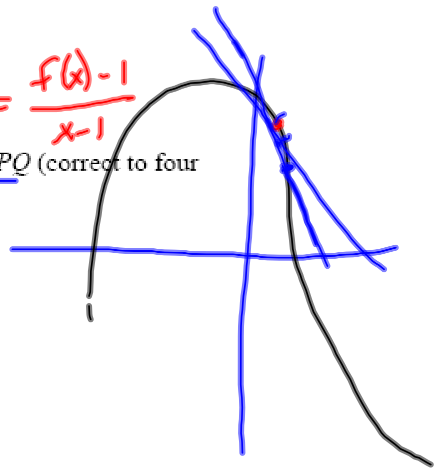
Math 131 Week in Review

Sections 2.1, 2.2, 2.3

2/7/10

$m = \frac{\Delta y}{\Delta x}$

$\frac{f(x) - f(1)}{x-1} = \frac{f(x)-1}{x-1}$



1. The point $P(1, 1)$ lies on the curve $y = \frac{2}{x+1}$.

   a. If Q is the point $\left(x, \frac{2}{x+1}\right)$, find the slope of the secant line PQ (correct to four decimal places) for the following values of x:

Δx	slope
0.5	$-.6667$
0.9	$-.5263$
0.99	$-.5025$
0.999	$-.5003$
1.001	$-.4998$
1.01	$-.4975$
1.1	$-.4762$
1.5	$-.4000$

   b. Using the results of part a, estimate the value of the slope of the tangent line to the curve at $P(1, 1)$.

   $m \approx -.5$

   c. Using the slope from part b, find an equation of the tangent line to the curve at $P(1, 1)$.

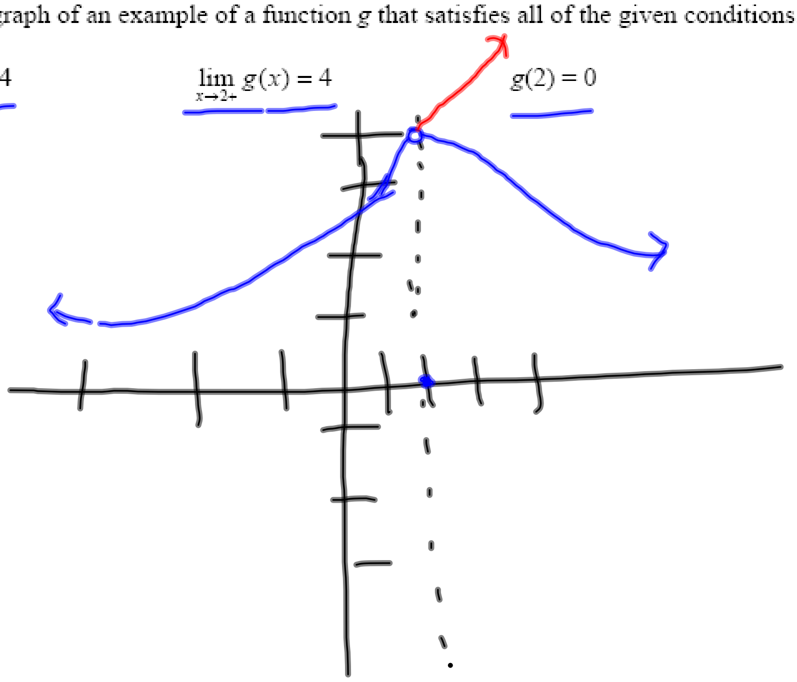
   $y - 1 = -.5(x-1)$

   $y - 1 = -.5x + .5$

   $y = -.5x + 1.5$

2. Sketch the graph of an example of a function g that satisfies all of the given conditions.

$\lim_{x\to 2-} g(x) = 4$ $\lim_{x\to 2+} g(x) = 4$ $g(2) = 0$



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3. Find the following limits:

a. $\lim_{x\to -2} 3x^3 - x^2 + 5$

$= 3(-2)^3 - (-2)^2 + 5$

$= 3(-8) - (4) + 5$

$= -24 - 4 + 5$

$= -23$

b. $\lim_{x\to 1} \dfrac{x^3 - x^2 + 3x - 3}{x-1}$

$x^2(x-1) + 3(x-1)$

$= \lim_{x\to 1} \dfrac{(x^2+3)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x\to 1} (x^2+3)$

$= 1^2 + 3$

$= 4$

c. $\lim_{x\to 3} \dfrac{x-3}{x+3}$

$= \dfrac{3-3}{3+3}$

$= \dfrac{0}{6}$

$= 0$

d. $\lim_{x\to 3} \dfrac{x^3-2}{x-3}$

DNE

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e. $\lim_{x\to 0}\dfrac{9-(x-3)^2}{x}$

$$= \lim_{x\to 0}\frac{9-(x^2-6x+9)}{x}$$

$$= \lim_{x\to 0}\frac{\cancel{9}-x^2+6x-\cancel{9}}{x}$$

$$= \lim_{x\to 0}\frac{-x^2+6x}{x} = \lim_{x\to 0}\frac{\cancel{x}(-x+6)}{\cancel{x}}$$

$$= \lim_{x\to 0}(-x+6)$$

$$= 6$$

f. $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}\cdot\dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$

$$= \lim_{x\to 0}\frac{(x+4)-4}{x(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0}\frac{\cancel{x}^{\,1}}{\cancel{x}(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}$$

$$= \frac{1}{\sqrt{0+4}+2}$$

$$= \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

g. $\lim_{x\to 0}\dfrac{\dfrac{1}{x+2}-\dfrac{2}{x}}{x}\cdot\dfrac{x(x+2)}{x(x+2)}$

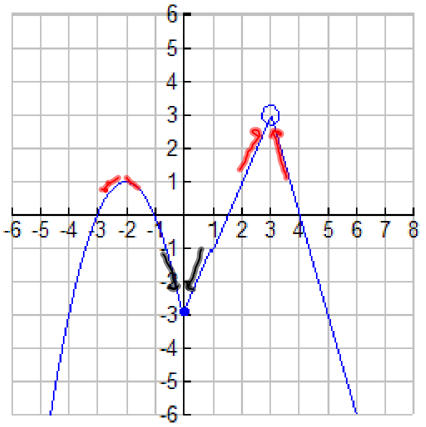
$$= \lim_{x\to 0}\frac{x-2(x+2)}{x^2(x+2)}$$

$$= \lim_{x\to 0}\frac{x-2x-4}{x^2(x+2)}$$

$$= \lim_{x\to 0}\frac{-x-4}{x^2(x+2)}$$

DNE

4. Use the graph of $g(x)$ below to find the indicated limits and function values.



a. $\lim_{x \to -2} g(x)$ = 1

b. $\lim_{x \to 0^-} g(x)$ = -3

c. $\lim_{x \to 0^+} g(x)$ = -3

d. $\lim_{x \to 0} g(x)$ = -3

e. $g(0)$ = -3

f. $\lim_{x \to 3^-} g(x)$ = 3

g. $\lim_{x \to 3^+} g(x)$ = 3

h. $\lim_{x \to 3} g(x)$ = 3

i. $g(3)$ DNE

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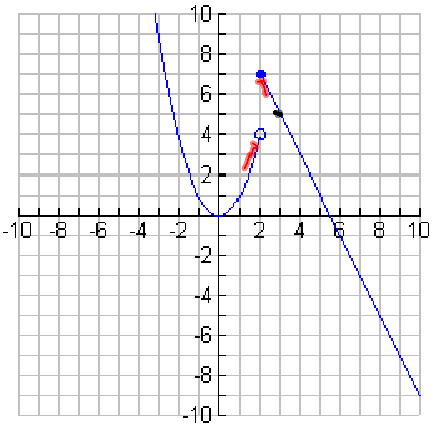
5. Use the graph of h below to find the indicated limits and function values.



a. $\lim_{x\to 3-} h(x) = 5$

b. $\lim_{x\to 3+} h(x) = 5$

c. $\lim_{x\to 3} h(x) = 5$

d. $h(3) = 5$

e. $\lim_{x\to 2-} h(x) = 4$

f. $\lim_{x\to 2+} h(x) = 7$

g. $\lim_{x\to 2} h(x)$ DNE

h. $h(2) = 7$

$6x^2 + 5x - 4$

$(3x + 4)(2x - 1)$

roots/zeroes $-\frac{4}{3}$ $\frac{1}{2}$

$(x + \frac{4}{3})(x - \frac{1}{2})$

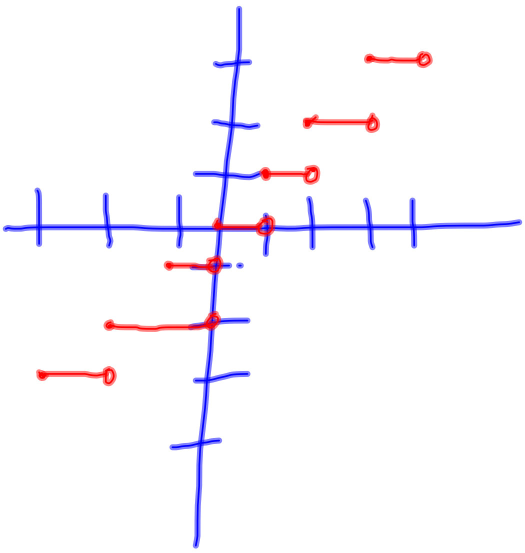
$(3x + 4)(2x - 1)$

x	$f(x)$
1	1
1.2	1
1.9	1
2	2
2.4	2
3	3
-1	-1
-1.2	-2
-1.9	-2
-2	-2
-2.1	-3

$f(x) = [x]$ $\quad [[x]]$

largest integer not greater than x

largest integer $\leq x$



$$\lim_{x \to -1} \frac{x^2+2x+1}{x^4-1}$$

$$= \lim_{x \to -1} \frac{(x+1)^2}{(x^2-1)(x^2+1)}$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x+1)}{\cancel{(x+1)}(x-1)(x^2+1)}$$

$$= \lim_{x \to -1} \frac{x+1}{(x-1)(x^2+1)}$$

$$= \frac{-1+1}{(-1-1)((-1)^2+1)}$$

$$= \frac{0}{(-2)(2)}$$

$$= \frac{0}{-4}$$

$$= 0$$

Graph $f(x)$ given

$\lim_{x \to 1^-} f(x) = 3$

$\lim_{x \to 1^+} f(x) = -1$

$f(1) = 2$