Math 131 Week in Review

Sections 1.1-1.3, 1.5-1.6, 2.1-2.5

2/14/2010

1. A cell phone company has a base charge of \$20 per month. The first 200 minutes are free, and the next 400 minutes cost \$0.15 per minute. Usage over 600 minutes costs \$0.25 per minute. Find a function, $C(t)$, the amount of a cell phone bill for the month in which a customer uses t cell phone minutes.

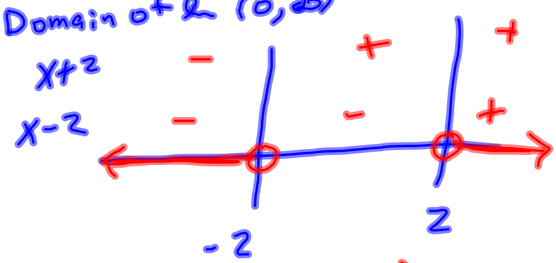
$$C(t) = \begin{cases} 20, & 0 \le t \le 200 \\ 20 + .15(t-200) & 200 < t \le 600 \\ 20 + .15(200) + .25(t-400) & t > 600 \end{cases}$$

2. Find the domain of $f(x) = \dfrac{3^{x-2}}{\ln \sqrt{x^2-4}}$.

Domain of ln $(0, \infty)$

$x^2 - 4 > 0$

$(x+2)(x-2) > 0$



$(-\infty, -2) \cup (2, \infty)$

3. The per capita consumption of potato chips in a small community increased in the past five years as shown in the table below. Let x = years after 2004

Year	2004 0	2005 1	2006 2	2007 3	2008 4
mg per person per year	2.1	2.4	2.6	2.9	3.2

Find a linear model through regression on the calculator. $y = .27x + 1.83$

a) Give the slope and interpret it in the context of the problem.

$m = .27$ mg/person/year increase in consumption of potato chips 2004-2008

b) If the trend continues, how many mg of potato will be expected per capita in 2011 (use the unrounded equation)?

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LinReg(ax+b) L1,
L2,Y1
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Y1(7)
            3.99
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3.99 mg per person in 2011

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LinReg
 y=ax+b
 a=.27
 b=2.1
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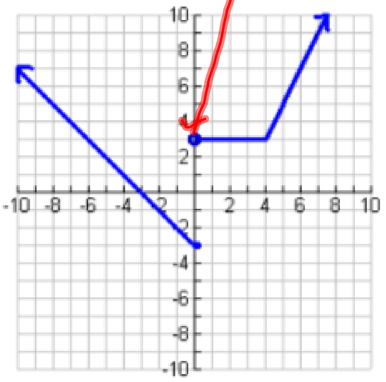
4. Find a function rule for the function f shown below.

should be open



3 linear pieces

$$f(x) = \begin{cases} -x-3, & x \le 0 \\ 3, & 0 < x \le 4 \\ 2x-5, & x > 4 \end{cases}$$

1st piece - slope $= -1$
y- intercept $(0, -3)$

2nd piece - horizontal
at $y = 3$

3rd piece - use 2 pts
$(4,3)$ $(6,7)$ $m = \frac{7-3}{6-4} = 2$

$y - 3 = 2(x-4)$
$y - 3 = 2x - 8$
$y = 2x - 5$

5. An object is launched into the air with a velocity of 80 feet per second, with height, h, after t seconds given by $h(t) = -16t^2 + 80t$ feet.

a) Find the height of the object after 3 seconds.

$h(3) = -16(3)^2 + 80(3) = -144 + 240 = 96$ ft

b) Find the average velocity between $t = 3$ and $t = 4$.

rate of change

$h(3) = 96$

$h(4) = -16(4)^2 + 80(4) = 64$

$\frac{64-96}{4-3} = -32$ ft/sec

c) Estimate the instantaneous velocity when $t = 3$ by calculating the average velocity over the following time intervals: [3, 3.01], [3, 3.001], and [3, 3.0001].

t	$h(t)$
3	96
3.01	95.8384
3.001	95.983984
3.0001	95.9983984

[3, 3.01] $\frac{95.8384-96}{3.01-3} = -16.16$

[3, 3.001] $\frac{95.983984-96}{3.001-3} = -16.016$

[3, 3.0001] $\frac{95.9983984-96}{3.0001-3} = -16.0016$

seems to be approaching -16 ft/sec

6. The half life of a certain radioactive substance is 350 years.

a) Find a function for the amount remaining after t years in a sample of size 80 grams.

$A = A_0 r^t$ $\quad A = 80\left(\frac{1}{2}\right)^{t/350}$

b) How much is left after 120 years? (Show algebraic ~~solution~~ setup)

80(1/2)^(12/35)
63.07825969

$A(120) = 80\left(\frac{1}{2}\right)^{\frac{120}{350}} \approx 63.078\text{ g}$

c) When will there be none of the substance left?

never, taking ½ over and over will never reach zero, just get smaller and smaller.

7. Evaluate $\lim_{x\to 3} \frac{2x^2 - x - 15}{x^2 - 9}$.

$= \lim_{x\to 3} \frac{(2x+5)\cancel{(x-3)}^{1}}{(x+3)\cancel{(x-3)}}$

$= \lim_{x\to 3} \frac{2x+5}{x+3}$

$= \frac{2(3)+5}{3+3}$

$= \frac{11}{9}$

Note: $\frac{(2x+5)(x-3)}{(x+3)(x-3)} \neq \frac{(2x+5)}{x+3}$

except when $x \neq 3$

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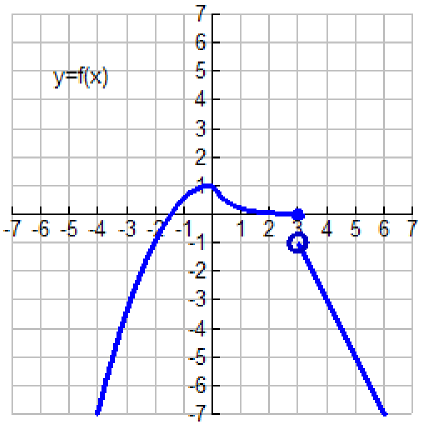
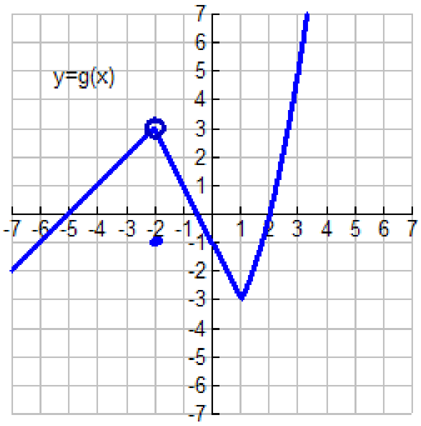
8. The graphs of f and g are shown below. Use them to evaluate each limit. If the limit does not exist, explain why.





a) $\lim\limits_{x\to 3}[f(x)+g(x)]$

$= \lim\limits_{x\to 3} f(x) + \lim\limits_{x\to 3} g(x)$

$= DNE + 4$ $\quad$ $\underline{DNE}$

b) $\lim\limits_{x\to 0}[f(x)+g(x)]$

$= \lim\limits_{x\to 0} f(x) + \lim\limits_{x\to 0} g(x)$

$= 1 + -1 = 0$

c) $\lim\limits_{x\to 0}[f(x)g(x)]$

$= \lim\limits_{x\to 0} f(x) * \lim\limits_{x\to 0} g(x)$

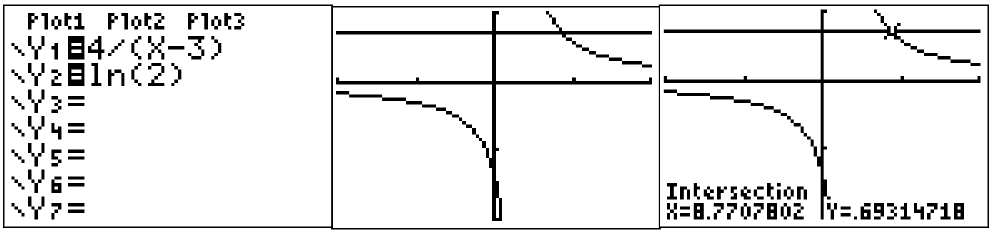
$= 1(-1) = -1$

d) $\lim\limits_{x\to -2}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to -2} f(x)}{\lim\limits_{x\to -2} g(x)} = \frac{-1}{3}$

9. If $f(x) = \sqrt{x-6}$, $g(x) = 3e^{2x}$, and $h(x) = \dfrac{2}{x-3}$, find $f \circ g \circ h$ and state its domain.

$f(g(h(x))) = f(g(\frac{2}{x-3})) = f(3e^{4/(x-3)}) = \sqrt{3e^{4/(x-3)}-6}$

$3e^{\frac{4}{x-3}} - 6 \geq 0 \Rightarrow 3e^{4/(x-3)} \geq 6 \Rightarrow e^{4/(x-3)} \geq 2 \Rightarrow \frac{4}{x-3} \geq \ln 2$



if multiply by $x-3$, is it pos or neg? Investigate on calculator

Domain: $(3, \ln 2)$

$3 \leq x \leq \ln 2$ $(\approx .69314718)$

9. If $f(x) = \sqrt{x-6}$, $g(x) = 3e^{2x}$, and $h(x) = \dfrac{2}{x-3}$, find $f \circ g \circ h$ and state its domain.

10. Write the function $g(y)$ which results from shifting the function $f(y) = e^y$ to the left 2 units, then reflecting about the x-axis, then reflecting about the y-axis, stretching vertically by a factor of four, and translating up 9 units.

$$g(y) = -4e^{-(y+2)} + 9$$

11. Find the values of a and b such that the function $f(x)$ is continuous for all real numbers x.

$$f(x)=\begin{cases} e^x & \text{if } x<0 \\ 2x-a & \text{if } 0\le x<2 \\ \cos x-b & \text{if } x\ge 2 \end{cases}$$

$e^0 = 2(0)-a$

$1 = -a$

$\boxed{a=-1}$

$2(2)-a = \cos 2 - b$

$4-a = \cos 2 - b$

$4-(-1) = \cos 2 - b$

$5 = \cos 2 - b$

$b+5 = \cos 2$

$\boxed{b = \cos 2 - 5}$

12. Find $\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}$. $\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$

$= \lim_{x\to 7} \frac{x+2-9}{(x-7)(\sqrt{x+2}+3)} = \lim_{x\to 7} \frac{\cancel{x-7}}{\cancel{(x-7)}(\sqrt{x+2}+3)}$

$= \lim_{x\to 7} \frac{1}{\sqrt{7+2}+3}$

$= \lim_{x\to 7} \frac{1}{3+3}$

$= \frac{1}{6}$

13. Find $\lim_{x\to\infty} \frac{x-2}{\sqrt{4x^2+1}}$ $= \lim_{x\to\infty} \frac{x}{2x} = \lim_{x\to\infty} \frac{1}{2} = \frac{1}{2}$

$$\lim_{x\to\infty} \frac{\frac{x}{\sqrt{x^2}} - \frac{2}{\sqrt{x^2}}}{\sqrt{\frac{4x^2}{x^2} + \frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2}$$

14. Solve for y: $2\log_2 y - \log_2(y-3) = \log_{10} 100$

$$\log_2 y^2 - \log_2 (y-3) = \log_{10} 10^2$$

$$\log_2 \frac{y^2}{y-3} = 2$$

$$2^2 = \frac{y^2}{y-3}$$

$$4(y-3) = y^2$$

$$0 = y^2 - 4y + 12$$

$$y = \frac{4 \pm \sqrt{16 - 4(12)}}{2}$$

$$= \frac{4 \pm \sqrt{-32}}{2}$$

no solution