Math 131 Week in Review
Sections 2.6, 2.7, 2.8, 3.1
2/28/10

$(a+h)(a+h)^2$
$=(a+h)(a^2+2ah+h^2)$
$=a^3+2a^2h+ah^2+a^2h+2ah^2+h^3$

Finding Derivatives By Definition (Sections 2.6-2.7)

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \checkmark$$

1. Find $f'(a)$ for $f(t)=t-2t^3$

$a+h \quad a+h \quad a+h$

$$\lim_{h\to 0}\frac{a+h-2(a+h)^3-(a-2a^3)}{h}$$
$$=\lim_{h\to 0}\frac{a+h-2(a^3+3a^2h+3ah^2+h^3)-(a-2a^3)}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{a}+h-\cancel{2a^3}-6a^2h-6ah^2-2h^3-\cancel{a}+\cancel{2a^3}}{h}$$
$$=\lim_{h\to 0}\frac{h-6a^2h-6ah^2-2h^3}{h}$$
$$=\lim_{h\to 0}\frac{\cancel{h}(1-6a^2-6ah-2h^2)}{\cancel{h}}$$
$$=\lim_{h\to 0}(1-6a^2-6ah-2h^2)$$
$$=\boxed{1-6a^2}$$

2. Find the equation to the tangent line of $f(t)$ in #1 at $x = 1$.

$f(1)=1-2(1)^3$
$=1-2$
$=-1$

$(1,-1)\quad m=-5$

$f'(1)=1-6(1)^2$
$=1-6$
$=-5$

$y-1=-5(x-1)$
$y+1=-5x+5$
$\boxed{y=-5x+4}$

3. Find $f'(a)$ for $f(x)=\frac{3}{x^2}$.

$$\lim_{h\to 0}\frac{\frac{3}{(a+h)^2}-\frac{3}{a^2}}{h}=\lim_{h\to 0}\frac{\frac{3a^2}{(a+h)^2a^2}-\frac{3(a+h)^2}{a^2(a+h)^2}}{h}$$
$$=\lim_{h\to 0}\frac{3a^2-3(a+h)^2}{a^2(a+h)^2}\cdot\frac{1}{h}$$
$$=\lim_{h\to 0}\frac{3a^2-3(a^2+2ah+h^2)}{a^2h(a+h)^2}$$
$$=\lim_{h\to 0}\frac{\cancel{3a^2}-\cancel{3a^2}-6ah-3h^2}{a^2h(a+h)^2}$$
$$=\lim_{h\to 0}\frac{-6ah-3h^2}{a^2h(a+h)^2}$$
$$=\lim_{h\to 0}\frac{\cancel{h}(-6a-3h)}{a^2\cancel{h}(a+h)^2}$$
$$=\lim_{h\to 0}\frac{-6a-3h}{a^2(a+h)^2}$$
$$=\frac{-6a-3(0)}{a^2(a+0)^2}=\frac{-6a}{a^4}=\boxed{\frac{-6}{a^3}}$$

4. Find $f'(a)$ for $f(x)=\dfrac{2}{x-3}$ $\lim_{h\to 0}\dfrac{\frac{2}{a+h-3}-\frac{2}{a-3}}{h} = \lim_{h\to 0}\dfrac{\frac{2(a-3)}{(a+h-3)(a-3)}-\frac{2(a+h-3)}{(a-3)(a+h-3)}}{h}$

$= \lim_{h\to 0}\dfrac{2(a-3)-2(a+h-3)}{(a+h-3)(a-3)}\cdot\dfrac{1}{h} = \lim_{h\to 0}\dfrac{\cancel{2a}\cancel{-6}\cancel{-2a}-2h\cancel{+6}}{h(a+h-3)(a-3)} = \lim_{h\to 0}\dfrac{-2\cancel{h}}{\cancel{h}(a+h-3)(a-3)}$

$= \lim_{h\to 0}\dfrac{-2}{(a+h-3)(a-3)}$

$= \boxed{\dfrac{-2}{(a-3)^2}}$

5. Find the equation to the tangent line for *f*(*x*) in #4 at *x* = 4.

$f(4)=\dfrac{2}{4-3}=\dfrac{2}{1}=2$ $f'(4)=\dfrac{-2}{(4-3)^2}$

$=\dfrac{-2}{1}$

$=-2$

point $(4,2)$ $m=-2$

$y-2=-2(x-4)$

$y-2=-2x+8$

$\boxed{y=-2x+10}$

6. Find $f'(a)$ for $f(x)=\frac{4}{\sqrt{1-x}}$. $\lim_{h\to 0}\dfrac{\frac{4}{\sqrt{1-a-h}}-\frac{4}{\sqrt{1-a}}}{h} = \lim_{h\to 0}\dfrac{\frac{4\sqrt{1-a}}{\sqrt{1-a-h}\sqrt{1-a}}-\frac{4\sqrt{1-a-h}}{\sqrt{1-a}\sqrt{1-a-h}}}{h}$

$= \lim_{h\to 0}\dfrac{4\sqrt{1-a}-4\sqrt{1-a-h}}{\sqrt{1-a-h}\sqrt{1-a}}\cdot\dfrac{1}{h} = \lim_{h\to 0}\dfrac{4(\sqrt{1-a}-\sqrt{1-a-h})}{h\sqrt{1-a-h}\sqrt{1-a}}\cdot\dfrac{\sqrt{1-a}+\sqrt{1-a-h}}{\sqrt{1-a}+\sqrt{1-a-h}}$

$= \lim_{h\to 0}\dfrac{4[\cancel{1}-\cancel{a}+(\cancel{-1}+\cancel{a}+h)]}{h\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})} = \lim_{h\to 0}\dfrac{4\cancel{h}^{1}}{\cancel{h}\sqrt{1-a-h}\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a-h})}$

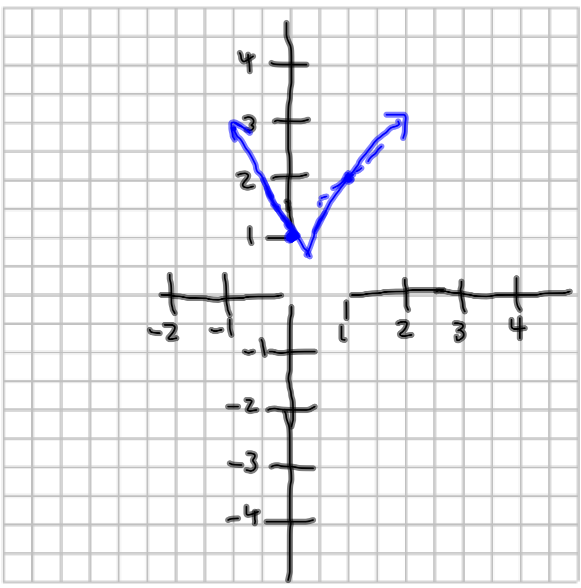
7. Sketch a graph of a function g for which $g(0) = 1$, $g'(0) = -2$, $g(1) = 2$, and $g'(1) = 1$.

$$= \frac{4}{\sqrt{1-a}\sqrt{1-a}(\sqrt{1-a}+\sqrt{1-a})}$$

$$= \frac{\cancel{4}^{2}}{|1-a|\cancel{2}\sqrt{1-a}}$$

$$= \frac{2}{|1-a|\sqrt{1-a}}$$

$\sqrt{x^2}=|x|$



position

8. A particle moves along a straight line with equation of motion $s(t) = 50t^0 + 8t^1 - 16t^2$.
Find the velocity and speed when $t = 3$.

$$s'(t) = 0(\ ) + 1\cdot 8\cdot t^0 - 2(16)t^1$$
$$= 0 + 8 - 32t$$

$$v(t) = s'(t) = 8 - 32t$$
$$v(3) = 8 - 32(3) = 8 - 96 = -88$$

Speed has no direction

$$|8 - 32t|$$
$$|8 - 32(3)| = |8 - 96| = |-88| = 88$$

Derivatives From a Table (Section 2.6)

9. The table below gives the approximate distance traveled by a downhill skier after t seconds for $0 \le t \le 10$. What is the meaning of $D'(6)$? Estimate its value.

meters distance

t	0	2	4	6	8	10
D	0	13	53	120	212	333

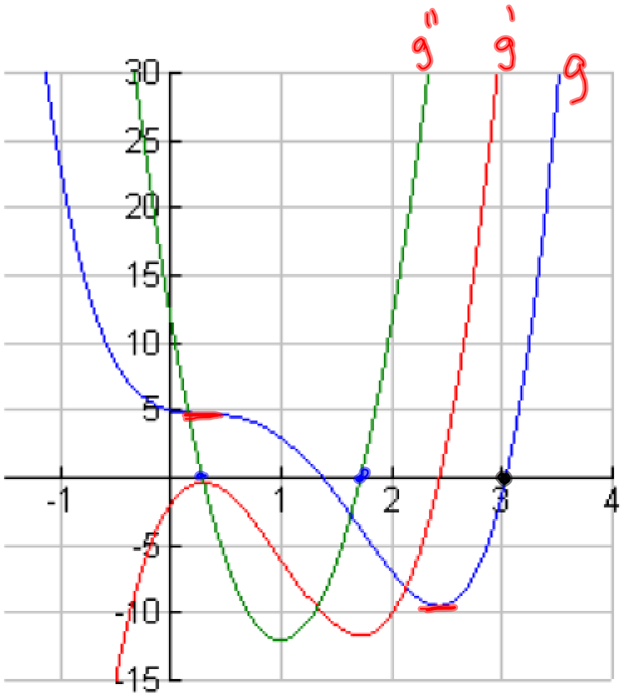
rate of change of distance over time (velocity) at 6 seconds

$\frac{120-53}{6-4} = \frac{67}{2}$ m/sec $\quad \frac{212-120}{8-6} = \frac{92}{2} = 46$ m/sec

$\frac{1}{2}\left(\frac{67}{2} + 46\right) = 39.75$ m/sec

10. The graphs of g, g', and g'' are given below. Label the graphs appropriately.

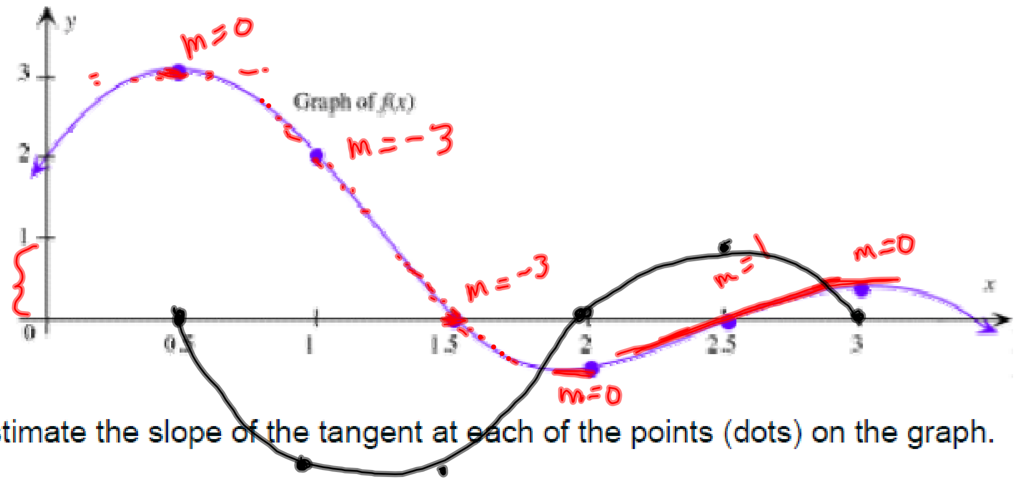
g' gives the slope of g



green is slope (derivative) of red

red is slope (derivative) of blue

Derivatives From a Graph (Section 2.7)



11. Estimate the slope of the tangent at each of the points (dots) on the graph.

12. Sketch the derivative of *f* on the same coordinate system above.

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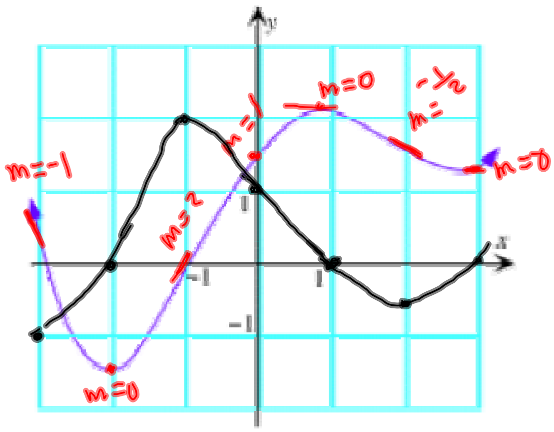


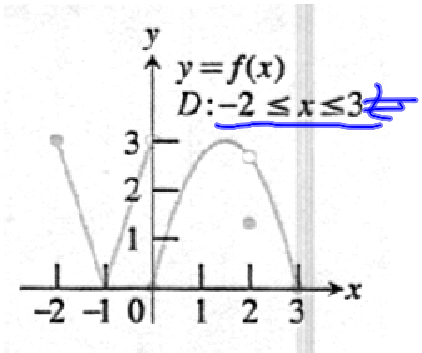
13. Estimate the slope of the tangent at each integer x-value on the graph.

14. Sketch the derivative of f on the same coordinate system above.

15. Give the intervals or points where the graph given below is not differentiable. Explain.

Finney, R. L., Demana, F. D., Waits, B. K., & Kennedy, D. (2007). *Calculus: Graphical, Numerical, Algebraic.*

Cusp (pointed)
discontinuous
vertical tangent line



$x = -1$ cusp
$x = 0$ discontinuous
$x = 2$ discontinuous
endpoint $x = -2$, $x = 3$

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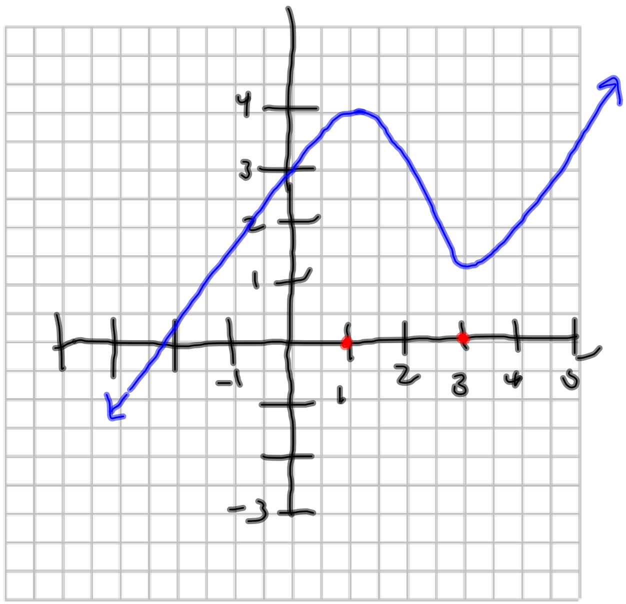
16. Sketch a graph of a function that satisfies all of the given conditions.

$F'(1) = F'(3) = 0$ horizontal tangent

$F'(x) > 0$ if $x < 1$ or $x > 3$ positive slope

$F'(x) < 0$ if $1 < x < 3$ negative slope



17. Differentiate: $g(x) = x\sqrt{29}$ $= \sqrt{29}\ x^1$

$g'(x) = 1\sqrt{29}\ x^0 = \sqrt{29}$

18. Differentiate: $f(t) = t^3 + 5t^2 - \frac{2}{3}t + 21$

$f'(t) = 3t^2 + 2(5)t^1 + 1(-\frac{2}{3})t^0 + 0$

$= 3t^2 + 10t - \frac{2}{3}$

19. Find any and all x-values where the tangent line is horizontal. on #18

slope $= 0$ $\quad f'(t) = 0$

$3t^2 + 10t - 2/3 = 0$

$9t^2 + 30t - 2 = 0$

$t = \frac{-30 \pm \sqrt{30^2 - 4(9)(-2)}}{2(9)}$

$= \frac{-30 \pm \sqrt{900 + 72}}{18}$

$= \frac{-30 \pm \sqrt{972}}{18}$

$= \frac{-30 \pm 18\sqrt{3}}{18}$

$= \frac{\cancel{6}(-5 \pm 3\sqrt{3})}{\cancel{6}(3)}$

$= \boxed{\frac{-5 \pm 3\sqrt{3}}{3}}$

$972 = 2^2 \cdot 3^5$

$= 2^2 \cdot 3^4 \cdot 3$

$\sqrt{2^2 \cdot 3^4 \cdot 3}$

$= 2 \cdot 3^2 \sqrt{3}$

$= 18\sqrt{3}$

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20. Differentiate: $h(x) = \frac{3x^2 - 6x + 24}{3x}$ $= \frac{3x^2}{3x} - \frac{6x}{3x} + \frac{24}{3x}$

$= x - 2 + 8x^{-1}$

$h'(x) = 1 + (-1)8x^{-2}$

$= 1 - 8x^{-2}$

$= 1 - \frac{8}{x^2}$ or $\frac{x^2 - 8}{x^2}$

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21. Differentiate: $f(u) = e^{u-1} + 3$

$\frac{d}{dx} e^{\wedge} = e^{\wedge}$

$f'(u) = e^{u-1}$

22. Find the equation of the tangent line for the function in #21, at $u = 1$.

$f'(1) = e^{1-1}$
$= e^{0}$
$= 1$

slope $= 1$

$f(1) = e^{1-1} + 3 = e^{0} + 3 = 1+3 = 4$

point $(1,4)$

$y - 4 = 1(x-1)$

$y - 4 = x - 1$

$\boxed{y = x + 3}$

23. Differentiate: $g(t) = \sqrt[3]{t} - 5\sqrt{t^3}$

$g(t) = t^{1/3} - 5t^{3/2}$

$g'(t) = \frac{1}{3}t^{-2/3} - \frac{3}{2}(5)t^{1/2}$

$= \frac{1}{3}t^{-2/3} - \frac{15}{2}t^{1/2}$?

$= \frac{1}{3\sqrt[3]{t^2}} \cdot \frac{\sqrt[3]{t}}{\sqrt[3]{t}} - \frac{15\sqrt{t}}{2}$?

$= \frac{\sqrt[3]{t}}{3t} - \frac{15\sqrt{t}}{2}$ ✓

$\frac{\sqrt[3]{t}\,(2)}{3t\,(2)} - \frac{15\sqrt{t}\,(3t)}{2(3t)}$

$\frac{2\sqrt[3]{t} - 45t\sqrt{t}}{6t}$ ✓