Math 131 Week in Review

Sections 3.2, 3.3, 3.4

3/7/2010

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x) = 1 \qquad \frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(e^x) = e^x$$

$$(f \pm g)' = f' \pm g' \qquad (cf)' = cf' \qquad (fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0 \qquad \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

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Differentiate the following functions:

1. $f(x) = \frac{1}{2}x^5 - 4x^3 + x - 2 + \underline{3/x}$ $3x^{-1}$

$$f'(x) = 5\left(\tfrac{1}{2}x^4\right) - 3(4)x^2 + 1 - 3x^{-2}$$
$$= \frac{5}{2}x^4 - 12x^2 + 1 - \frac{3}{x^2}$$

2. $g(t) = \dfrac{t^3 + t - 4}{2t - 3}$

$$g'(t) = \frac{(2t-3)(3t^2+1) - (t^3+t-4)(2)}{(2t-3)^2}$$
$$= \frac{6t^3 - 9t^2 + \cancel{2t} - 3 - 2t^3 - \cancel{2t} + 8}{(2t-3)^2}$$
$$= \frac{4t^3 - 9t^2 + 5}{(2t-3)^2}$$

3. $h(x) = \sqrt{5x} - \sqrt[3]{2x^2}$

$$= (5x)^{1/2} - (2x^2)^{1/3}$$
$$= \frac{1}{2}(5x)^{-1/2}(5) - \frac{1}{3}(2x^2)^{-2/3}(4x)$$
$$= \frac{5}{2\sqrt{5x}} - \frac{4x}{3\sqrt[3]{(2x^2)}} \quad \text{unsimplified}$$
$$= \frac{5}{2\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}} - \frac{4x}{3\sqrt[3]{(2x)^2}} \cdot \frac{\sqrt[3]{2x}}{\sqrt[3]{2x}}$$
$$= \frac{\cancel{5}^{1}\sqrt{5x}}{2(\cancel{5}x)} - \frac{\cancel{4}^{2}\cancel{x}^{1}\sqrt[3]{2x}}{3(\cancel{2x})}$$
$$= \frac{\sqrt{5x}}{2x} - \frac{2\sqrt[3]{2x}}{3} \quad \text{rationalized + simplified}$$

$$\frac{\sqrt{5x}}{2x} \cdot \frac{3}{3} - \frac{2\sqrt[3]{2x}}{3} \cdot \frac{2x}{2x}$$
$$\frac{3\sqrt{5x} - 4x\sqrt[3]{2x}}{6x}$$
Common denominator

4. $y = 3e^x - 2x^3 + 4$

$$y' = 3e^x - 6x^2$$

5. $f(t) = (t + 2e^t)(5 - \sqrt{t}\,)$ $\quad t^{1/2}$

$$f'(t) = (t + 2e^t)\left(-\tfrac{1}{2}t^{-1/2}\right) + (1+2e^t)(5-\sqrt{t})$$

$$= -\tfrac{1}{2}t^{1/2} + \underline{2t^{-1/2}e^t} + 5 - \sqrt{t} + 10e^t - 2e^t\sqrt{t}$$

$$\frac{2e^t}{\sqrt{t}} = \frac{2e^t\sqrt{t}}{t}$$

6. $h(x) = \dfrac{x - \sin x}{1 + \cos x}$

$$h'(x) = \frac{(1+\cos x)(1-\cos x) - (x - \sin x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{1 - \cos^2 x - (-x\sin x + \sin^2 x)}{(1+\cos x)^2}$$

$$= \frac{1 - \underline{\cos^2 x} + x\sin x \underline{- \sin^2 x}}{(1+\cos x)^2} \qquad \sin^2 x + \cos^2 x = 1$$

$$= \frac{\cancel{1-1}^{\,0} + x\sin x}{(1+\cos x)^2}$$

$$= \frac{x\sin x}{(1+\cos x)^2}$$

7. $g(x) = 5 \sec x - 3 \tan x$

$g'(x) = 5 \sec x \tan x - 3 \sec^2 x$

$= \sec x (5 \tan x - 3 \sec x)$

8. $F(t) = (2te^t)(\cot t)$

$F'(t) = 2te^t(-\csc^2 t) + (2te^t + 2e^t)\cot t$

$= -2te^t \csc^2 t + 2te^t \cot t + 2e^t \cot t$

9. $f(x) = (2x^3 + 2)^4$

$$f'(x) = 4(2x^3+2)^3(6x^2)$$
$$= 24x^2(2x^3+2)^3$$

Chain Rule

$(f\circ g)(x)$

$= f(\underline{g(x)})$

$(f\circ g)'(x)$

$= f'(\underline{g(x)})\,\underline{g'(x)}$

10. $g(x) = \underline{\cos}\ (\underline{x^2 - x})$

$$g'(x) = \underline{-\sin}(\underline{x^2-x})\ (2x-1)$$
$$= -(2x-1)\sin(x^2-x)$$

11. $G(t) = \overset{f}{\tan}\ (4t - \overset{g}{\ }\sin 3t)$

$$G'(t) = \sec^2(4t-\sin 3t)\left[(4 - \cos 3t\ \ (3))\right]$$
$$= (4-3\cos 3t)\sec^2(4t-\sin 3t)$$

12. $H(x) = (1 + \cos 2x)^2$

$$H'(x) = 2(1+\cos 2x)'(-\sin 2x)(2)$$

$$= -4\sin 2x\,(1+\cos 2x) \checkmark$$

$$\sin 2x = 2\sin x\cos x$$

$$= -4\sin 2x - 4\sin 2x\cos 2x \checkmark$$

$$= -4\sin 2x - 2(2\sin 2x\cos 2x)$$

$$= -4\sin 2x - 2\sin 4x \checkmark$$

13. Find y'' for $y = \sec x$

$$y' = \sec x\tan x$$

$$y'' = \sec x(\sec^2 x) + (\sec x\tan x)\tan x$$

$$= \sec x(\sec^2 x + \tan^2 x) \quad \text{or} \quad \sec^3 x + \sec x\tan^2 x$$

perpendicular to tangent line

14. Find an equation of the tangent line and normal line to $y = (2 - 3x)^2$ at $x = -1$.

tangent

$y - 25 = -30(x+1)$

$y - 25 = -30x - 30$

$y = -30x - 5$

normal

$y - 25 = \frac{1}{30}(x+1)$

$y - 25 = \frac{1}{30}x + \frac{1}{30}$

$y = \frac{1}{30}x + \frac{751}{30}$

slope of normal line $= -\frac{1}{30}$

$y'(-1) = -30$ slope of tangent

$y(-1) = (2+3)^2 = 25$

point $(-1, 25)$

$y' = 2(2-3x)(-3)$

$= -6(2-3x)$

$= -12 + 18x$

15. The equation of motion of a particle is $s(t) = t^3 - 2t^2 + t - 4$, where s is in feet and t is in seconds. Find

a) the velocity and acceleration as functions of t.

$v(t) = s'(t) = 3t^2 - 4t + 1$

$a(t) = v'(t) = s''(t) = 6t - 4$

b) the acceleration after 2 seconds.

$a(2) = 6(2) - 4 = 12 - 4 = 8 \text{ ft/sec}^2$

c) The acceleration when the velocity is 0.

need t

$0 = 3t^2 - 4t + 1$

$0 = (3t - 1)(t - 1)$

$t = \frac{1}{3} \quad t = 1$

$a(\frac{1}{3}) = 6(\frac{1}{3}) - 4 = 2 - 4 = -2 \text{ ft/sec}^2$

$a(1) = 6(1) - 4 = 2 \text{ ft/sec}^2$

16. For what value(s) of x does the graph of $F(x) = 2x^3 + x^2 - 4x - 5$ have a horizontal tangent?

slope = 0

$F'(x) = 6x^2 + 2x - 4$

$6x^2 + 2x - 4 = 0$

$3x^2 + x - 2 = 0$

$(3x - 2)(x + 1) = 0$

$x = 2/3 \qquad x = -1$

17. Find an equation of the tangent line to $G(x) = 2x - \sqrt{x}$ that is parallel to the line $y = 1 - 3x$.

same slope

$m = {}^{-}3$

$G'(x) = 2 - \frac{1}{2}x^{-1/2}$

$= 2 - \frac{1}{2\sqrt{x}}$

$.10 - .02 = .08$

point $(.01, -.08)$

$2 - \frac{1}{2\sqrt{x}} = -3$

$-\frac{1}{2\sqrt{x}} = -5$

$\frac{1}{2\sqrt{x}} = 5$

$1 = 5(2\sqrt{x})$

$1 = 10\sqrt{x}$

$(.1)^2 = (\sqrt{x})^2$

$.01 = x$ x-value for point where tangent line has slope = -3

$G(.01) = 2(.01) - \sqrt{.01}$

$= .02 - .1$

$= -.08$

$y + .08 = -3(x - .01)$

$y + .08 = -3x + .03$

$\boxed{y = -3x - .05}$

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18. For $f(t) = x^2 - 2x - 3 + 1/x$, find $f''(-2)$.

$f'(t) = 2x - 2 - x^{-2} \Longrightarrow f''(t) = 2 + 2x^{-3}$

$= 2x - 2 - \frac{1}{x^2}$

or $2 + \frac{2}{x^3}$

or $\frac{2x^3+2}{x^3}$

19. Find an equation of the tangent line to $y = 2x \cos x$ at $x = \pi/2$.

$y(\frac{\pi}{2}) = 2\frac{\pi}{2}\cos\frac{\pi}{2}$

$= \pi(0)$

$= 0$

point $(\frac{\pi}{2}, 0)$

$y' = 2x(-\sin x) + 2\cos x$

$= -2x\sin x + 2\cos x$

$y'(\frac{\pi}{2}) = -2\frac{\pi}{2}\sin\frac{\pi}{2} + 2\cos\frac{\pi}{2}$

$= -\pi(1) + 2(0)$

$= -\pi$ slope

$y - 0 = -\pi(x - \frac{\pi}{2})$

$\boxed{y = -\pi x + \frac{\pi^2}{2}}$

20. Given $f(5) = 0$, $f'(5) = -1$, $g(5) = 3$, and $g'(5) = 2$, find $(fg)'(5)$, $(f/g)'(5)$, and $(g/f)'(5)$.

$(fg)'(5) = f(5)g'(5) + f'(5)g(5)$

$= 0(2) + (-1)(3)$

$= -3$

$\left(\frac{f}{g}\right)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2}$

$= \frac{3(-1) - 0(2)}{3^2}$

$= \frac{-3}{9}$

$= -\frac{1}{3}$

$\left(\frac{g}{f}\right)'(5) = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2}$

$= \frac{0(2) - 3(-1)}{0^2}$ does not exist

Since $f(5) = 0$

$\frac{g}{f}(5)$ is undefined

21. Given $h(x) = f(g(x))$ and the table of data below, find $h'(4)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	4	2	-2	0
4	4	3	0	1
5	2	-4	-3	5

$h'(x) = f'(g(x))\,g'(x)$

$h'(4) = f'(g(4))\,g'(4)$

$= f'(3)\,g'(4)$

$= -2(1)$

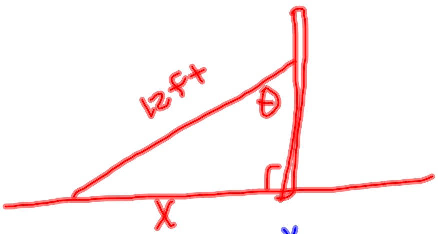
$= -2$

22. An object moves along the x-axis so that its position at any time $t \geq 0$ is given by $s(t) = \sin(t^2 + 1)$. Find the velocity of the object as a function of t.

$v(t) = s'(t) = \cos(t^2+1)\ (2t)$

$v(t) = 2t\cos(t^2+1)$

23. A ladder 12 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall, and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?



Find $\frac{dx}{d\theta}$ when $\theta = \frac{\pi}{3}$

$\sin\theta = \frac{x}{12}$

$x = 12\sin\theta$

$\frac{d}{d\theta}\ 12\sin\theta$

how fast (rate) x change as θ change

$= 12\cos\theta$

when $\theta = \frac{\pi}{3}$

$12\cos\frac{\pi}{3} = 12\left(\frac{1}{2}\right) = 6$ ft/rad

For every 1 radian increase in θ,
x increases 6 ft
(ladder slides out 6 ft)