Math 131 Week in Review

Sections 2.6 - 2.8, 3.1 - 3.4, 3.7 – 3.9

3/21/10

1. Use the definition to find the derivative of $f(x) = \dfrac{2x-3}{x+1}$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{2(x+h)-3}{x+h+1}\cdot\frac{(x+1)}{(x+1)} - \frac{2x-3}{x+1}\cdot\frac{(x+h+1)}{(x+h+1)}}{h}$$

$$= \lim_{h\to 0}\frac{(2x+2h-3)(x+1) - (2x-3)(x+h+1)}{(x+h+1)(x+1)}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{2x^2+2x+2xh+2h-3x-3-(2x^2+2xh+2x-3x-3h-3)}{(x+h+1)(x+1)h}$$

$$= \lim_{h\to 0}\frac{\cancel{2x^2}+\cancel{2x}+\cancel{2xh}+2h-\cancel{3x}-\cancel{3}-\cancel{2x^2}-\cancel{2xh}-\cancel{2x}+\cancel{3x}+3h+\cancel{3}}{(x+h+1)(x+1)h}$$

$$= \lim_{h\to 0}\frac{5\cancel{h}}{(x+h+1)(x+1)\cancel{h}}$$

$$= \frac{5}{(x+1)(x+1)}$$

$$= \frac{5}{(x+1)^2}$$

2. Find the equation of the tangent line and the normal line to the following curve when $t = 0$.

$$y = (3t^2 - 2t + 1)(t - \cos t)$$

$\perp$ to tangent line

tangent line

point $(0,-1)$

$m = 3$

$y - -1 = 3(x - 0)$

$y + 1 = 3x$

$\boxed{y = 3x - 1}$

point

$y = (0 - 0 + 1)(0 - \cos 0)$

$= -1\cos 0$

$= -1(1)$

$= -1$

$(0,-1)$

slope

$y' = (3t^2 - 2t + 1)(1 + \sin t) + (6t - 2)(t - \cos t)$

$= 3t^2 + 3t^2\sin t - 2t - 2t\sin t + 1 + \sin t + 6t^2 - 6t\cos t - 2t + 2\cos t$

$= 9t^2 + 3t^2\sin t - 4t - 2t\sin t + 1 + \sin t - 6t\cos t + 2\cos t$

$y'(0) = \cancel{0 + 0 - 0 - 0} + 1 + \sin 0 - \cancel{0} + 2\cos 0$

$= 1 + 0 + 2(1)$

$= 3 \qquad m = 3$ tangent line

normal line

point $(0,-1)$

$m = -1/3$

$y - -1 = -1/3(x - 0)$

$y + 1 = -1/3\, x$

$\boxed{y = -\frac{1}{3}x - 1}$

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3. A piece function is defined as follows:

$\frac{x+2}{x-3}$ not continuous @ $x=3$

$$F(x)=\begin{cases} x+1 & x \le 0 \\ e^x & 0 \le x < 1 \\ 2x^2-1 & x \ge 1 \end{cases}$$

$x \to 0^-$ $x \to 0^+$ $x \to 1^-$ $x \to 1^+$

a) Find any intervals where F is discontinuous.

$\lim_{x\to 0^-} F(x) = \lim_{x\to 0^-} (x+1) = 1$ $\lim_{x \to 0^+} F(x) = \lim_{x\to 0^+} e^x = e^0 = 1$ continuous

b) Find any intervals where F is not differentiable.

$\lim_{x\to 1^-} F(x) = \lim_{x\to 1^-} e^x = e$

$\lim_{x\to 1^+} F(x) = \lim_{x\to 1^+} (2x^2-1) = 1$

discontinuous at $x=1$

✱ not differentiable @ $x=1$ because it is discontinuous

$$F'(x) = \begin{cases} 1 & x < 0 \\ e^x & 0 \le x < 1 \\ 4x & x \ge 1 \end{cases}$$

$\lim_{x\to 0^-} F'(x) = \lim_{x\to 0^-} 1 = 1$

$\lim_{x\to 0^+} F'(x) = \lim_{x\to 0^+} e^x = e^0 = 1$

differentiable at $x=0$

differentiability
not if
1) discontinuous
2) cusp
3) vertical tangent line

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4. Find the derivative of the following functions:

a) $G(x) = \tan(e^x - 2x)$

$$G'(x) = \sec^2(e^x - 2x)\,(e^x - 2)$$
$$= (e^x - 2)\sec^2(e^x - 2x)$$

b) $H(x) = \dfrac{e^{-2x} + e}{e^{2x} - e^{-x} + 2}$

$$H'(x) = \frac{(e^{2x} - e^{-x} + 2)\left(e^{-2x}(-2) + 0\right) - (e^{-2x} + e)\left(e^{2x}(2) - e^{-x}(-1)\right)}{(e^{2x} - e^{-x} + 2)^2}$$
$$= \frac{(e^{2x} - e^{-x} + 2)(-2e^{-2x}) - (e^{-2x} + e)(2e^{2x} + e^{-x})}{(e^{2x} - e^{-x} + 2)^2}$$
$$= \frac{-2e^{0} + 2e^{-3x} - 4e^{-2x} - \left(2e^{0} + e^{-3x} + 2e^{2x+1} + e^{-x+1}\right)}{(e^{2x} - e^{-x} + 2)^2}$$
$$= \frac{-2 + 2e^{-3x} - 4e^{-2x} - 2 - e^{-3x} - 2e^{2x+1} - e^{-x+1}}{(e^{2x} - e^{-x} + 2)^2}$$
$$= \frac{-4 + e^{-3x} - 4e^{-2x} - 2e^{2x+1} - e^{-x+1}}{(e^{2x} - e^{-x} + 2)^2}$$

c) $F(x) = \ln(2x^3 - 5)^2$

$$F'(x) = \frac{1}{(2x^3 - 5)^2}\,2(2x^3 - 5)'(6x^2)$$
$$= \frac{12x^2\cancel{(2x^3 - 5)}}{(2x^3 - 5)^{\cancel{2}\,1}}$$
$$= \frac{12x^2}{2x^3 - 5}$$

5. The temperature (in degrees Fahrenheit) of a person during an illness can be modeled by $T = -0.0375t^2 + 0.3t + 100.4$, where t is time in hours since the person started to show signs of a fever.
Larson, R. (2009). *Applied Calculus for the Life and Social Sciences*.

a) Find dT/dt and explain its meaning in this situation.

$T'(t) = -.075t + .3$

rate of change of temperature per hour at time t

b) Find the rate of change of body temperature after 4 hours.

$T'(4) = -.075(4) + .3 = 0$

temperature is not changing 4 hours after fever began

c) Over what interval(s) is the function increasing?

$T'(t) > 0$ $\quad -.075t + .3 > 0 \Rightarrow -.075t > -.3$

$t < 4$ $\quad (0, 4)$

d) Over what interval(s) is the function concave down?

$T''(t) < 0$

$T''(t) = -.075$ $\quad (0, \infty)$

6. Where does the function given below have a horizontal tangent line or lines over the interval [0, 2π)?

slope = 0

What x-values

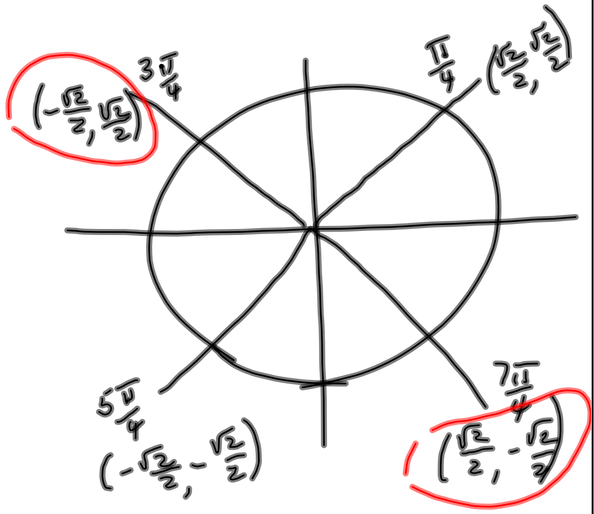
$F(x) = \cos x - \sin x$

$F'(x) = -\sin x - \cos x$

$-\sin x - \cos x = 0$

$-\sin x = \cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$



7. The function $g(t)=\dfrac{t^2-t+1}{t^2+1}$ measures the level of oxygen in a pond, where t is the time (in weeks) after organic waste is dumped into the pond.
Larson, R. (2009). *Applied Calculus for the Life and Social Sciences.*

Find the rate of change of ~~f~~ g with respect to t when $t = .5$.

$$g'(t) = \frac{(t^2+1)(2t-1)-(t^2-t+1)(2t)}{(t^2+1)^2}$$

$$= \frac{2t^3-t^2+2t-1-(2t^3-2t^2+2t)}{(t^2+1)^2}$$

$$= \frac{\cancel{2t^3}-t^2\cancel{+2t}-1-\cancel{2t^3}+2t^2\cancel{-2t}}{(t^2+1)^2}$$

$$= \frac{t^2-1}{(t^2+1)^2}$$

$$g'(.5) = \frac{(.5)^2-1}{(.5^2+1)^2} = \frac{.25-1}{(1.25)^2} = -\frac{12}{25} \text{ or } -.48$$

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8. The predicted cost C (in hundreds of thousands of dollars) for a company to remove $p\%$ of a chemical from its waste water can be modeled by

$$C = \frac{124p}{(10+p)(100-p)}, \quad 0 \le p < 100.$$

Larson, R. (2009). *Applied Calculus for the Life and Social Sciences.*

a) Find the rate of change in the cost when $p = 25\%$.

$$C'(25) = \frac{124(1000+25^2)}{(10+25)^2(100-25)^2} = \frac{124(1625)}{35^2(75)^2} \approx .0292$$

b) Find the rate of change in the cost when $p = 75\%$.

$$C'(75) = \frac{124(1000+75^2)}{(10+75)^2(100-75)^2} = \frac{124(6625)}{85^2 25^2} \approx .1819$$

c) What happens to C and C' as p approaches 100?

$\lim_{p\to 100} C = \infty$ $\quad$ $\lim_{p\to 100} C' = \infty$ $\quad$ rate of change is increasing faster + faster

$$C'(p) = \frac{(10+p)(100-p)(124) - 124p[(10+p)(-1) + 1(100-p)]}{(10+p)^2(100-p)^2}$$

$$= \frac{124[(10+p)(100-p) - p(-10-p+100-p)]}{(10+p)^2(100-p)^2}$$

$$= \frac{124[1000+90p-p^2+10p+p^2-100p+p^2]}{(10+p)^2(100-p)^2}$$

$$= \frac{124[1000+p^2]}{(10+p)^2(100-p)^2}$$

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9. An environmental study indicates that the average daily level P of a certain pollutant in the air (in parts per million) can be modeled by the equation

$$P = 0.25\sqrt{0.5n^2 + 5n + 25}$$

where n is the number of residents of the community (in thousands). Find the rate at which the level of pollutant is increasing when the population of the community is 12,000. $n = 12$

Larson, R. (2009). *Applied Calculus for the Life and Social Sciences.*

$$P'(n) = .25\left(\tfrac{1}{2}\right)(.5n^2+5n+25)^{-1/2}(1n+5)$$

$$= \frac{n+5}{8\sqrt{.5n^2+5n+25}}$$

$$P'(12) = \frac{12+5}{8\sqrt{.5(12)^2+5(12)+25}} = \frac{17}{8\sqrt{157}} \approx .170$$

.170 parts/million rate of increase of pollutant when there are 12,000 people

10. The number of children enrolled E (in thousands per year) in the State Children's Health Insurance Program (SCHIP) for 1998 through 2005 can be modeled by $E = \frac{21{,}892 - 2812.9t}{1 - 0.2759t}$, where t is the time in years, with $t = 8$ corresponding to 1998.

Larson, R. (2009). *Applied Calculus for the Life and Social Sciences.*

$t=10$ $\quad t=15$

a) Find the average rate of change from 2000 through 2005.

$$E(10) = \frac{21892 - 28129}{1 - 2.759} \approx 3545.765$$

$$E(15) = \frac{21892 - 2812.9(15)}{1 - .2759(15)} \approx 6468.536$$

$$\frac{E(15) - E(10)}{15 - 10} = 584.552$$

about 584552 children per year were enrolled from 2000 - 2005

b) Find the instantaneous rate of change for 2005.

$t=15$

$$E'(t) = \frac{(1 - .2759t)(-2812.9) - (21892 - 2812.9)(-.2759)}{(1 - .2759t)^2}$$

$$= \frac{-2812.9 + \cancel{776.07911t} + 6040.0028 - \cancel{776.07991t}}{(1 - .2759t)^2}$$

$$= \frac{3227.1028}{(1 - .2759t)^2}$$

$$E'(15) = \frac{3227.1028}{(1 - .2759 * 15)^2} \approx 327.619$$

about 327,619 children per year in 2005

11. The normal daily maximum temperature T (in degrees Fahrenheit) for a Midwest city can be modeled by $T = \sqrt{5.9918t^4 - 171.015t^3 + 1469.25t^2 - 3560.6t + 4292}$, where t is the time in months, with $t = 1$ corresponding to January.

Larson, R. (2009). *Applied Calculus for the Life and Social Sciences.*

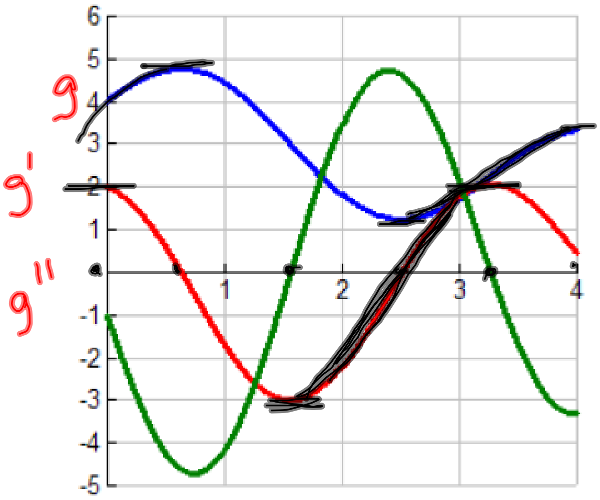
a) Find the rate of change in March. $t = 3$

$$T'(t) = \frac{1}{2}\left(5.9918t^4 - 171.015t^3 + 1469.25t^2 - 3560.6t + 4292\right)^{-1/2}\left(23.9672t^3 - 513.045t^2 + 2938.5t - 3560.6\right)$$

$T'(3) \approx 12.358°F$ increase in max daily temperature per month in March

12. The following is a graph of a function *g*, along with its first and second derivative. Identify which is g, g ′, and *g* ″.

blue red green



when slope of blue = 0, red has value of 0

red is blue' ?

when blue has m > 0, is red's value pos ? (above x axis).

Yes! red = blue'

when slope of red = 0, green has value of 0

green is red' ?

When red has pos. slope, is green's value pos (above x axis) ?

Yes! green = red'

13. Estimate the derivative of f at $x = 2$, given the table below.

average 2 slopes

x	0	1	2	3	4
y	5.5	2.3	4.1	3.5	5.1

$$m_1 = \frac{4.1-2.3}{2-1} = 1.8$$

$$m_2 = \frac{3.5-4.1}{3-2} = -.6$$

$$\frac{m_1+m_2}{2} = \frac{1.8-.6}{2} = \frac{1.2}{2} = .6$$

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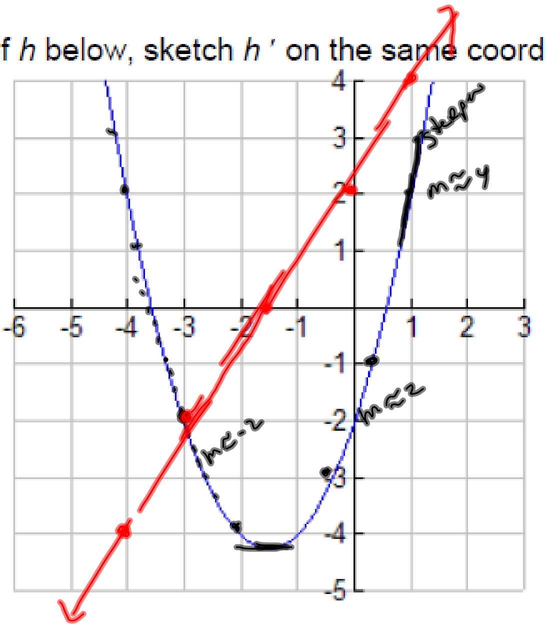
14. Given the graph of h below, sketch h' on the same coordinate system.

$\frac{2}{\frac{1}{2}} = 4$

$m \approx -4$



15. Given the position function $s(t) = x^3 - 2.5x^2 - 2x - 4.2$, find the acceleration when the velocity is zero.

$v(t) = 0$
$s'(t) = 0$

$$v(t) = s'(t) = 3x^2 - 5x - 2$$

$$3x^2 - 5x - 2 = 0$$
$$(3x + 1)(x - 2) = 0$$
$$\cancel{x = -1/3} \qquad x = 2$$

time ≥ 0

$a(2)$?

$$a(t) = v'(t) = 6t - 5$$
$$a(2) = 6(2) - 5$$
$$= 12 - 5$$
$$= 7$$

use units if given

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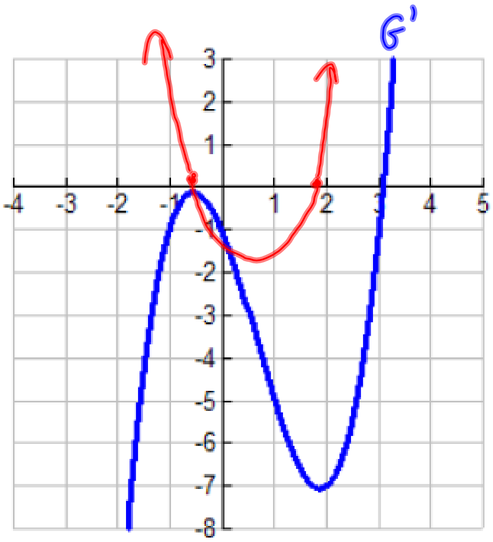
16. Given the graph of G' below, give the intervals where G is increasing, decreasing, concave up, and concave down.



G is inc where $G' > 0$

$(3, \infty)$ dec $(-\infty, 3)$

G is CU where $G'' > 0$

CU $(-\infty, -\frac{1}{2})$, $(1.9, \infty)$

CD $(-\frac{1}{2}, 1.9)$

17. Given $f(x) = \frac{1}{1-x}$, find the linear approximation near the point $a = 0$.

$f'(x) = \frac{(1-x)(0) - 1(-1)}{(1-x)^2}$

$= \frac{1}{(1-x)^2}$

$f'(0) = \frac{1}{(1-0)^2} = 1$

$f(0) = \frac{1}{1-0} = 1$

$y = f(a) + f'(a)(x-a)$

$y = 1 + 1(x-0)$

$\boxed{y = x+1}$

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