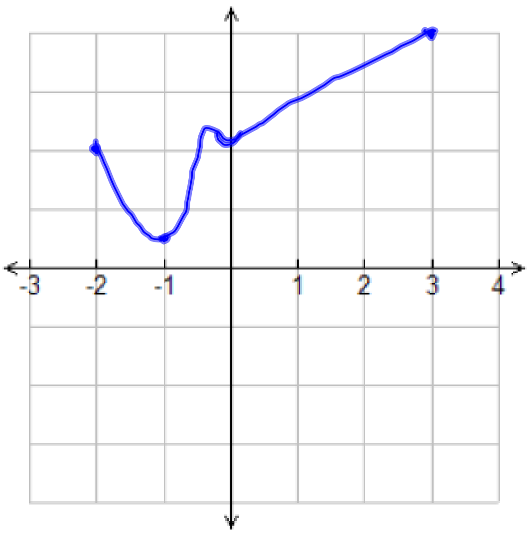
Math 131 Week in Review
Sections 4.2, 4.3, 4.6
4/4/10

1. Sketch a graph of a function *f* that is continuous on [-2, 3], has an absolute minimum at -1, an absolute maximum at 3, and a local minimum at 0.



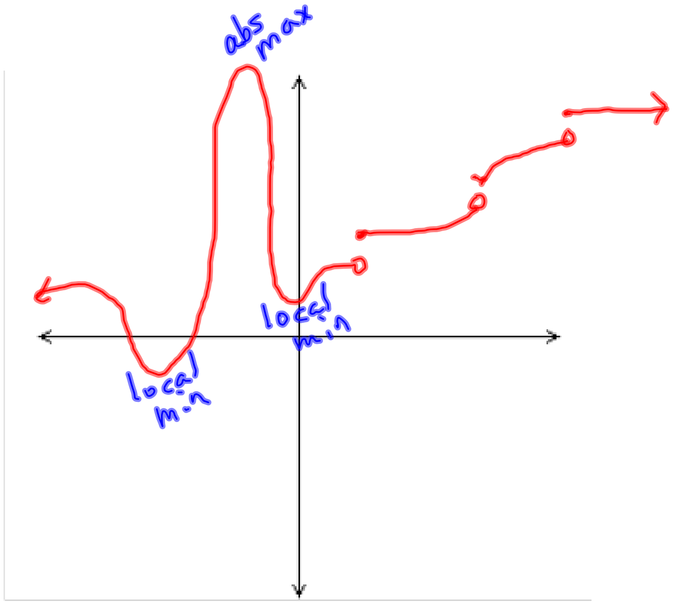
A critical number of a function *f* is a number *c* in the domain of *f* such that either $f'(c) = 0$ or $f'(c)$ does not exist.

horizontal tangent line

f is discontinuous
cusp
vertical tangent line

2. Sketch the graph of a function that has 2 local minima, 1 absolute maximum, and 6 critical numbers.

3 more



3. Find the critical numbers of $g(x) = 2\cos x - \sin^2 x,\ 0 \le x < \pi$.

$g'(x) = -2\sin x - 2\sin x \cos x$ def. on D

$-2\sin x - 2\sin x\cos x = 0$

$-2\sin x(1+\cos x) = 0$

$-2\sin x = 0$ $\quad 1+\cos x = 0$

$\sin x = 0$ $\quad \cos x = -1$

$\boxed{x=0}$ $\quad x \ne \pi$ not in domain

4. Find the critical numbers of $h(x) = x^3 - 3x^2 - 4x$.

def on $\mathbb{R}$

$h'(x) = 3x^2 - 6x - 4$

$3x^2 - 6x - 4 = 0$

$x = \dfrac{6 \pm \sqrt{36-4(3)(-4)}}{2(3)}$

$= \dfrac{6 \pm \sqrt{84}}{6}$

$= \dfrac{6 \pm 2\sqrt{21}}{6}$

$= \dfrac{\cancel{2}^{1}(3 \pm \sqrt{21})}{\cancel{6}\,3}$

$= \boxed{\dfrac{3 \pm \sqrt{21}}{3}}$

$2\,\lfloor 84$
$2\,\lfloor 42$
21

5. Find the critical numbers of $F(x) = x^2 e^{-4x}$.

def on $\mathbb{R}$

$F'(x) = x^2 e^{-4x}(-4) + 2xe^{-4x}$

$= -4x^2e^{-4x} + 2xe^{-4x}$

$= -2xe^{-4x}(2x-1)$

$\underset{\ne 0}{-2}\, x\, \underset{\ne 0}{e^{-4x}}(2x-1) = 0$

$\boxed{x=0}$ $\quad 2x-1=0$

$\boxed{x=\frac{1}{2}}$

6. Find the critical numbers of $G(x) = x^{\frac{2}{3}}(x-3)^2$.

def on $\mathbb{R}$

$$G'(x) = x^{2/3}(2)(x-3) + \frac{2}{3}x^{-1/3}(x-3)^2$$
$$= x^{-1/3}(x-3)\left[2x + \frac{2}{3}(x-3)\right]$$
$$= x^{-1/3}(x-3)\left(\frac{8}{3}x - 2\right)$$

$x^{-1/3} \to \frac{1}{\sqrt[3]{x}}$

$x^{-1/3}(x-3)\left(\frac{8}{3}x-2\right) = 0$

$x-3=0$, $x=3$

$\frac{8}{3}x - 2 = 0$, $\frac{8}{3}x = 2$, $8x = 6$, $x = \frac{6}{8} = \frac{3}{4}$

$$\boxed{x=0 \quad x=3,\ x=\frac{3}{4}}$$

7. Find the absolute maximum and absolute minimum value of $f(x) = x^4 - 3x^2 + 2$ on the interval [-2, 3].

def on $\mathbb{R}$

$f'(x) = 4x^3 - 6x$

$4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

$x = 0$ $\quad 2x^2 - 3 = 0$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}} = \frac{\pm\sqrt{6}}{2}$

$f(-2) = (-2)^4 - 3(-2)^2 + 2 = 6$

$f(0) = 0 - 0 + 2 = 2$

$f\left(-\frac{\sqrt{6}}{2}\right) = \frac{9}{4} - 3\left(\frac{3}{2}\right) + 2 = -\frac{1}{4}$ abs min

$f\left(\frac{\sqrt{6}}{2}\right) = \frac{9}{4} - 3\left(\frac{3}{2}\right) + 2 = -\frac{1}{4}$

$f(3) = 81 - 3(9) + 2 = 56$ abs max

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8. Find the absolute maximum and absolute minimum value of $G(x) = \frac{x^2-9}{x^2+9}$ on the interval [-5, 5].

$$G'(x) = \frac{(x^2+9)(2x)-(x^2-9)(2x)}{(x^2+9)^2}$$

def everywhere on [-5,5]

$$= \frac{2x(x^2+9-x^2+9)}{(x+9)^2}$$

$$= \frac{2x(18)}{(x+9)^2}$$

$$= \frac{36x}{(x+9)^2}$$

$\frac{36x}{(x+9)^2} = 0$ when $36x = 0$

$x = 0$

$G(-5) = \frac{25-9}{25+9} = \frac{16}{34} = \frac{8}{17}$

$G(0) = \frac{-9}{9} = -1$

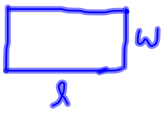
$G(5) = \frac{8}{17}$

abs min $= -1$

abs max $= \frac{8}{17}$

9. What is the smallest perimeter possible for a rectangle whose area is 16 in^2, and what are its dimensions?

Finney, Demana, Waits, and Kennedy, *Calculus: Graphical, Numerical, Algebraic*, 3rd ed., 2007.



minimize

$P = 2w + 2l$

$A = wl$

$wl = 16$

$l = \frac{16}{w}$

$P(w) = 2w + 2\left(\frac{16}{w}\right)$

$= 2w + 32w^{-1}$

$P'(w) = 2 - 32w^{-2}$

$= 2 - \frac{32}{w^2}$

$2 - \frac{32}{w^2} = 0$

$2w^2 - 32 = 0$

$2(w^2-16) = 0$

$2(w+4)(w-4) = 0$

$\cancel{w = -4}$ $\quad w = 4$ in

$l = \frac{16}{4} = 4$ in

4 in x 4 in

$P = 2(4) + 2(4) = 16$ in.

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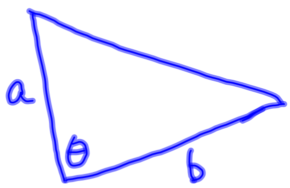
10. Two sides of a triangle have lengths a and b, and the angle between them is θ. What value of θ maximize the triangle's area? [Hint: $A = \frac{1}{2} ab \sin\theta$.]

Finney, Demana, Waits, and Kennedy, *Calculus: Graphical, Numerical, Algebraic, 3rd ed.*, 2007.



$0 < \theta < 180$

$A = \frac{1}{2} ab \sin\theta$

$A'(\theta) = \frac{1}{2} ab \cos\theta$

$\frac{1}{2} ab \cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^\circ, \cancel{270^\circ}, \ldots$

11. The height of an object moving vertically is given by $s = -16t^2 + 96t + 112$, with s in ft and t in sec.

Finney, Demana, Waits, and Kennedy, *Calculus: Graphical, Numerical, Algebraic*, 3rd ed., 2007.

Find

i. the object's velocity when $t = 0$,

$$v(t) = s'(t) = -32t + 96$$
$$v(0) = -32(0) + 96 = 96 \text{ ft/sec}$$

ii. its maximum height and when it occurs, and

$$v(t) = -32t + 96$$
$$-32t + 96 = 0$$
$$32t = 96$$
$$\boxed{t = 3 \text{ sec}}$$

$$s(3) = -16(3)^2 + 96(3) + 112$$
$$= -144 + 288 + 112$$
$$= \boxed{256 \text{ ft}}$$

iii. its velocity when $s = 0$.

$$-16t^2 + 96t + 112 = 0$$
$$-16(t^2 - 6t - 7) = 0$$
$$-16(t - 7)(t + 1) = 0$$
$$t = 7 \text{ sec} \quad \cancel{t = -1 \text{ sec}}$$

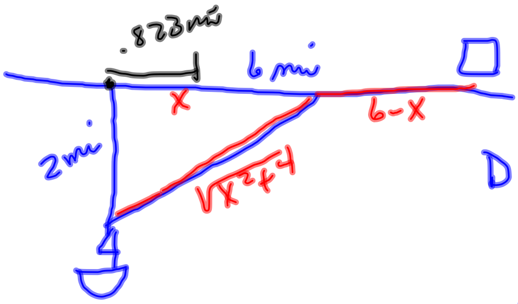
$$v(7) = -32(7) + 96$$
$$= -224 + 96$$
$$= \boxed{-128 \text{ ft/sec}}$$

12. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

Finney, Demana, Waits, and Kennedy, *Calculus: Graphical, Numerical, Algebraic*, 3rd ed., 2007.

$D = r \cdot t$

$t = \frac{D}{r}$

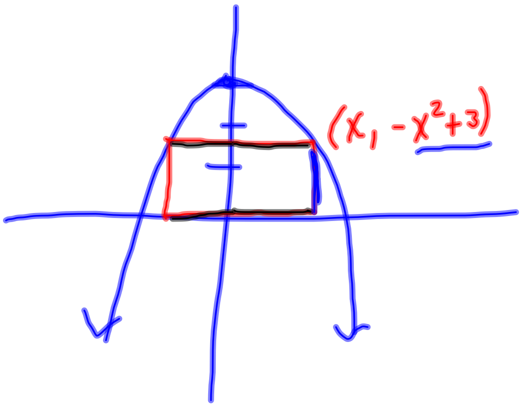


$\left(6 - \frac{4}{\sqrt{21}}\right)$ mi from village

$$D(x) = \frac{\sqrt{x^2+4}}{2} + \frac{6-x}{5}$$
$$= \frac{1}{2}\sqrt{x^2+4} + \frac{6}{5} - \frac{1}{5}x$$
$$D'(x) = \frac{1}{2}\left(\frac{1}{2}\right)(x^2+4)^{-1/2}(2x) - \frac{1}{5}$$
$$= \frac{x}{2\sqrt{x^2+4}} - \frac{1}{5}$$

$$\frac{x}{2\sqrt{x^2+4}} - \frac{1}{5} = 0$$
$$\frac{x}{2\sqrt{x^2+4}} = \frac{1}{5}$$
$$5x = 2\sqrt{x^2+4}$$
$$25x^2 = 4(x^2+4)$$
$$25x^2 = 4x^2+16$$
$$21x^2 = 16$$
$$x^2 = \frac{16}{21}$$
$$x = \pm\frac{4}{\sqrt{21}}$$
$$x = \frac{4}{\sqrt{21}} \approx .873 \text{ mi}$$

14. A rectangle is to be inscribed on the x-axis under the arch of the curve $y = -x^2 + 3$. What are the dimensions of the rectangle with largest area, and what is the largest area?



$A = wl$

$A(x) = 2x(-x^2+3)$

$= -2x^3 + 6x$

maximize:

$A'(x) = -6x^2 + 6$

$-6x^2+6=0$

$-6(x^2-1)=0$

$-6(x+1)(x-1)=0$

$x \neq -1 \quad x = 1$

$y = -(1)^2+3 = 2$

width $= 2x = 2(1) = 2$

length $= -x^2+3 = 2$

Area $= 2(2) = 4$

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