Math 131 Week in Review

Sections 4.8, 5.1, 5.2

4/11/10

1. Find the most general antiderivative of $f(x) = 72x^{-1.1}$.

$$F(x) = \frac{72\, x^{-1.1+1}}{(-1.1+1)} + C = \frac{72x^{-.1}}{-.1} + C = -720x^{-.1} + C$$

2. Find the most general antiderivative of $g(r) = (4r + 9)^2$. $= 16r^2 + 72r + 81$

$$G(r) = \frac{16r^3}{3} + \frac{72r^2}{2} + 81r + C$$

$$= \frac{16}{3}r^3 + 36r^2 + 81r + C$$

3. Find the most general antiderivative of $h(t) = \sec t \tan t$.

$H'(t) = \sec t + C$

4. Find the most general antiderivative of $f(x) = 3e^x + 5\sqrt[3]{x^2} - \sqrt[5]{x^3}$.

$f(x) = 3e^x + 5x^{2/3} - x^{3/5}$

$F(x) = 3e^x + \frac{5x^{5/3}}{\frac{5}{3}} - \frac{x^{8/5}}{\frac{8}{5}} + C$

$= 3e^x + \frac{3}{5}(5)x^{5/3} - \frac{5}{8}x^{8/5} + C$

$= 3e^x + 3x^{5/3} - \frac{5}{8}x^{8/5} + C$

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x^c

5. Find the most general antiderivative of $g(x) = \dfrac{3x^5 - 2x^3 + 5x}{x^4}$ $= 3x - 2\cancel{x^{-1}} + 5x^{-3}$ $\frac{1}{x}$

$$G'(x) = \frac{3x^2}{2} - 2\ln x + \frac{5x^{-2}}{-2} + C$$

$$= \frac{3}{2}x^2 - 2\ln x - \frac{5}{2}x^{-2} + C$$

6. Find f given $f'(t) = 1 + \sin t$ and $f(0) = 2$.

$$f(t) = t - \cos t + C$$

$$\boxed{f(t) = t - \cos t + 1}$$

$f(0) = 2$

$f(0) = 0 - \cos 0 + C$

$2 = 0 - \cos 0 + C$

$2 = 1 + C$

$C = 1$

7. Find g given $g''(x) = 6 - e^{-x}$, $g(0) = -6$, and $g(-1) = -e - 2$.

$$g'(x) = 6x + e^{-x} + C_1$$

$$g(x) = \frac{6x^2}{2} - e^{-x} + C_1x + C_2$$

$$* \quad g(x) = 3x^2 - e^{-x} + C_1x + C_2$$

$$-6 = 3(0)^2 - e^{-0} + C_1(0) + C_2$$

$$-6 = 0 - 1 + C_2$$

$$C_2 = -5$$

$$g(x) = 3x^2 - e^{-x} + C_1x - 5$$

$$-e - 2 = 3(-1)^2 - e^{--1} + C_1(-1) - 5$$

$$-e - 2 = 3 - e - C_1 - 5$$

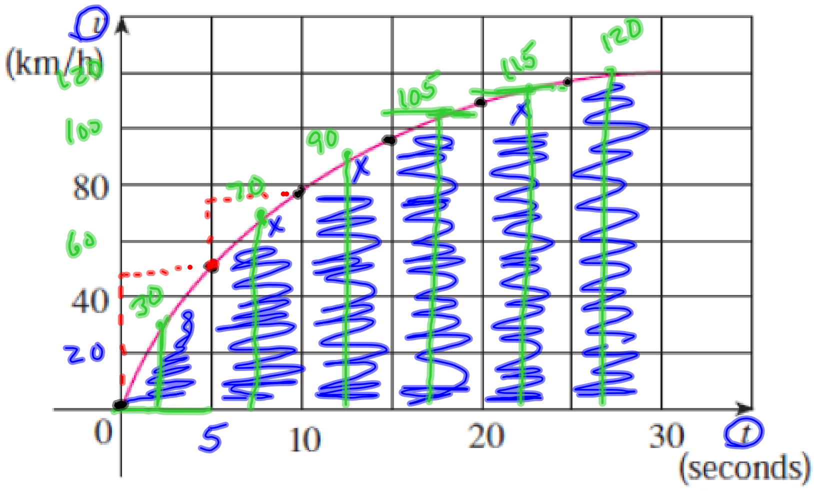
$$\cancel{-e}\cancel{-2} = \cancel{-2}\cancel{-e} - C_1$$

$$C_1 = 0$$

$$g(x) = 3x^2 - e^{-x} + 0x - 5$$

$$\boxed{g(x) = 3x^2 - e^{-x} - 5}$$

8. The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.
(p. 342 #16)



$25\frac{1}{2}$ rectangles

100 km /rectangle

25.2(100)

= 2520 km

$5(30+70+90+105+115+120) = 5(530) = 2650$ km

9. Jim walked along the bank of a tidal river watching the incoming tide carry a bottle upstream, recording the velocity of the flow every 5 minutes for 30 minutes. The results are shown in the table below.
Adapted from Finney, Demana, Waits, & Kennedy, *Calculus: Graphical, Numerical, Algebraic*, 2007.

D = rt

Time (min)	Velocity (m/sec)
0	1
5	1.2
10	1.7
15	2.0
20	1.8
25	1.6
30	1.4

About how far upstream does the bottle travel during the half-hour? Find L_6 and R_6.

$L_6 = 5(1+1.2+1.7+2+1.8+1.6) = 5(9.3) = 46.5$ meters

$R_6 = 5(1.2)+5(1.7)+5(2.0)+5(1.8)+5(1.6)+5(1.4)$

$= 9.7(5) = 48.5$ meters

10. Express the limit as a definite integral on the interval [-3, 4].

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\sin x_i}{x_i}\Delta x$$

dx

$$\int_{-3}^{4}\frac{\sin x}{x}\,dx$$

11. Use the form of the definition of the integral given below to evaluate the integral

$$\int_{-2}^{1}(1-2x)\,dx$$

$f(x)=1-2x$

Definition: If f is integrable on [a, b], then $\int_a^b f(x)dx=\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x$ where $\Delta x=\frac{b-a}{n}$

and $x_i = a + i\Delta x$.

$a=-2$

$\Delta x = \frac{1--2}{n} = \frac{3}{n}$

$x_i = -2 + i\left(\frac{3}{n}\right)$

$= -2 + \frac{3i}{n}$ ✓

$$\int_{-1}^{2}(1-2x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[1-2\left(-2+\frac{3i}{n}\right)\right]\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[1+4-\frac{6i}{n}\right]\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left[5-\frac{6i}{n}\right]\frac{3}{n}$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[\sum_{i=1}^{n}5-\sum_{i=1}^{n}\frac{6i}{n}\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[\boxed{\sum_{i=1}^{n}5}-\frac{6}{n}\sum_{i=1}^{n}i\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[5n-\frac{6}{\cancel{n}}\cdot\frac{\cancel{n}(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[5n-\frac{6n+6}{2}\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[5n-3n-3\right]$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[2n-3\right]$$

$$= \lim_{n\to\infty}\left(6-\frac{9}{n}\right)$$

$= 6$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} c = cn$

$\int_{-2}^{1}(1-2x)dx$

$= x - \frac{2x^2}{2}\Big]_{-2}^{1}$

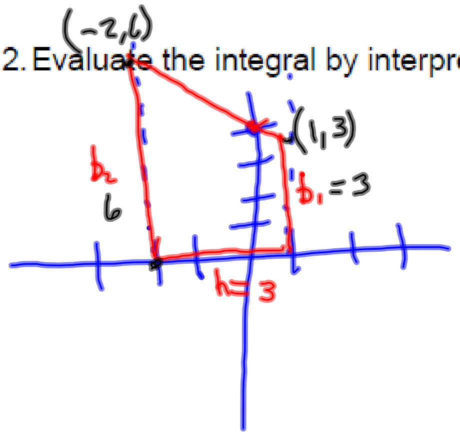
$= x - x^2\Big]_{-2}^{1}$

$= 1-1^2-\left(-2-(-2)^2\right)$

$= 2+4$

$= 6$

12. Evaluate the integral by interpreting it in terms of areas: $\int_{-2}^{1} |4-x|\,dx$.



$4--2=6$

$4-1=3$

Area of trapezoid:

$A=\frac{b_1+b_2}{2}h$

$=\frac{3+6}{2}(3)$

$=\frac{27}{2}$

$=13.5$ sq units

13. Use the properties of integrals to evaluate $\int_{-2}^{1} (4x-3x^2)\,dx$.

$\frac{4x^2}{2}-\frac{3x^3}{3}\Big]_{-2}^{1}$

$=2x^2-x^3\Big]_{-2}^{1}$

$=2(1)-1^3-\left[2(-2)^2-(-2)^3\right]$

$=2-1-[8--8]$

$=2-1-8-8$

$=-15$

14. Given that $\int_{-2}^{11} f(x)\,dx = 32$ and $\int_{-2}^{0} f(x)\,dx = 17$, find $\int_{0}^{11} f(x)\,dx$.

$$\int_{-2}^{11} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{11} f(x)\,dx$$

$$32 = 17 + \int_{0}^{11} f(x)\,dx$$

$$15 = \int_{0}^{11} f(x)\,dx$$

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