Polytope Exercises

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Exercise 1. Let $A \subset \mathbb{R}^n$ be a finite set, and let $P = \text{conv}(A)$ be a polytope. For $w \in \mathbb{R}^n$, define

$$A_w := A \cap P_w,$$

where P_w is the face exposed by w. Prove that

$$A_w = \{a \in A \mid w^T a = h_p(w)\}.$$

Exercise 2. Using the notation from the previous exercise, prove that $P_w = \text{conv}(A_w)$.

Exercise 3. Let $f \in \mathbb{C}[x_1^\pm, \ldots, x_n^\pm]$, and let $A = \text{supp}(f)$. For any $w \in \mathbb{R}^n$, we have a partial order on $\mathbb{C}[x_1^\pm, \ldots, x_n^\pm]$ given by:

$$x^a <_w x^b \quad \text{if} \quad w^T a < w^T b.$$

Show that $\text{supp}(\text{in}_w f) = A_w$.