Spring 2012 - Math 131
Exam 2 Review

courtesy: Kendra Kilmer
(coversing Sections 2.6-2.8, 3.1-3.4, 3.7-3.8)

Please note that this is not an all-inclusive review. This is just a sampling of problems from these sections. To work more problems from these sections, please see WEB5-80945.

1. If \( f(x) = \sqrt{2x + 5} \), use the limit definition of the derivative to find the slope of the tangent line at \( x = 4 \).

2. The number of people at a festival \( t \) hours after it has opened is given by \( N(t) \). Interpret the following statements.
   (a) \( N(3) = 400 \)
   (b) \( N'(3) = 100 \)
   (c) \( N''(3) = 5 \)

3. Use the limit definition of the derivative to find \( f'(x) \) if \( f(x) = \frac{4x^2 + 3}{2 + x} \).

4. Given the graph of \( f(x) \) below sketch the graph of \( f'(x) \).

5. Given the graphs below identify \( f, f' \) and \( f'' \).

6. Given the graph below,

   (a) If the graph is of \( f \) determine where \( f \) is increasing, decreasing, concave up, concave down. Also determine where \( f \) has local extrema and inflection points.

7. Use the given information to sketch a graph of \( f \).
   - Domain: \( (-\infty, -6) \cup (-6, \infty) \)
   - Vertical Asymptote at \( x = -6 \)
   - \( \lim_{x \to -6^+} f(x) = -2 \)
   - \( f(-10) = 3, f(-2) = 0, f(1) = -7, f(5) = -10, f(-8) = -7 \)
   - \( f'(x) > 0 \) on \( (-6, -2) \cup (5, \infty) \)
   - \( f''(x) < 0 \) on \( (-\infty, -6) \cup (-2, 5) \)
   - \( f''(x) > 0 \) on \( (-\infty, -10) \cup (1, 8) \)
   - \( f''(x) < 0 \) on \( (-10, -6) \cup (-6, 1) \cup (8, \infty) \)

8. Differentiate the following functions.
   (a) \( f(x) = \frac{(4x^2 - 7x + 2)^8}{x^7} \)
   (b) \( y = (x^2 + 1) \cdot \cos(5x) \cdot \sin(2x + 5) \)
   (c) \( g(x) = \log_2 \left( \frac{(10x^2 - 2x)^5}{(4x^3 - x)^3} \right) \)
   (d) \( y = \tan(\ln(\sqrt{x^2 + 1})) \)

9. Find the equation of the tangent line to the curve \( y = \sqrt{32x^2 + \ln[(x - 3)^2]} \) at \( x = 4 \).

10. Find a second-degree polynomial \( P \) such that \( P(1) = 24, P'(1) = 11, P''(1) = 8 \).

11. Find the exact values of \( x \) for which the tangent line to the curve of \( y = 3x^2 - \frac{1}{2}x^2 - 8x + 100 \) is horizontal.

12. If \( f(x) \) is a differentiable function, find an expression for the derivative of each of the following functions:
   (a) \( g(x) = \frac{[4f(x) + x^2]^8}{f(x) + 2x} \)
   (b) \( h(x) = (5f(x) + 1)^2 \cdot (4x^3 - x) \cdot (4f(x)) \)

13. The position of a particle is given by the function \( s(t) = t^4 - \frac{12}{5}t^3 + 12t \) for \( t \geq 0 \) where \( t \) is measured in seconds and \( s \) is in feet.
   (a) Find the velocity at time \( t \).
   (b) When is the particle at rest?
   (c) When is the particle moving in the positive direction? When is the particle moving in the negative direction?
   (d) Draw a diagram to represent the motion of the particle and find the total distance traveled by the particle during the first six seconds.
   (e) Find the acceleration at time \( t \).
   (f) When is the particle speeding up? When is it slowing down?

14. Find \( y' \) and \( y'' \) for
   (a) \( y = 12x^9 + 9x + 15 \)
   (b) \( y = \frac{\ln(5x)}{x^5} \)