

## Peano's Axioms of Arithmetic

Giuseppe Peano was born August 27, 1858 in Cuneo, Piemonte, Italy, and died April 20, 1932 in Turin, Italy. During his lifetime he accomplished much. One amazing result of his is a space filling curve. That is, a continuous map of the interval  $[0, 1]$  onto the unit square in  $R^2$ . A second accomplishment, and one relevant to this course, is his axiomatization of arithmetic. The following axioms, due to him, can be used to construct the integers, the rational numbers, and eventually the real numbers. While we will not have the time to do all of the work necessary for these constructions, we will see some of the trees in this forest during the semester.

1. There is a set  $S$ , and an element,  $\lambda$ , such that  $\lambda \in S$ .
2. For any  $a \in S$  there is another unique element in  $S$  denoted by

$$a + .$$

The element  $a+$  is called the successor of  $a$ , and  $a$  is called the precursor of  $a+$ .

3. For no  $b \in S$  is it true that  $b+ = \lambda$ .
  4. If  $a+ = b+$  for any  $a$  and  $b$  in  $S$ , then  $a = b$ .
  5. If  $P$  is any subset of  $S$  that has the following properties:
    - $\lambda \in P$ ,
    - if  $x \in P$ , then  $x+ \in P$ ,
    - then  $P = S$ .
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Here is a question. Is it possible for a set that satisfies Peano's axioms to have two different elements that satisfy property 3? That is, is there an  $a \in S$  besides  $\lambda$  such that  $b+ = a$  is not true for any  $b \in S$ ? Said another way; Is  $\lambda$  the only element in  $S$  that has no precursor? Write up your solution, and turn it in the next class period.