March 9, 1995 Math 304, Exam 1

(35) 1. Let $A = \begin{bmatrix} 2 & 2 & 0 & -4 & 6 \\ 5 & 5 & -1 & -11 & 14 \\ 2 & 2 & 0 & -4 & 3 \\ 10 & 10 & -3 & -23 & 27 \end{bmatrix}$.

a. Find a basis for the null space of A.

b. Find a basis for the column space of A.

c. Describe the solution set of the equation $A\vec{x} = \vec{b}$, for $\vec{b} = (8, 19, 2, 37)$.

d. Describe the solution set of the equation $A\vec{x} = \vec{b}$, for $\vec{b} = (1, 1, 1, 1)$.

(10) 2. Let $\{ \vec{x}_1, \vec{x}_2 \}$, be two vectors in R^{10}. Let $\vec{y}$ be a nonzero vector such that $\vec{y}$ is perpendicular to each of the first two vectors. Show that $\vec{y}$ is perpendicular to every vector in the span of $\{ \vec{x}_1, \vec{x}_2 \}$.

(30) 3. Let $B = \{ 1 + t, \ t^2 - 3, \ t + t^3, \ 2t - 1 \}$. Let $S = \{ 1, \ t, \ t^2, \ t^3 \}$.

a. Show that B is a basis of the vector space P_3.

b. Let $\vec{p}$ be vector in P_3 such that its coordinates with respect to the basis B are $[-1, 2, 3, 1]$. What does $\vec{p}$ equal.

c. Find the change of basis matrix Q, where $[\vec{p}]_B = Q[\vec{p}]_S$.

(15) 4. Let $V = \{ \vec{x} \in R^4 : < \vec{x}, (1, 0, 1, 0) >= 0 =< \vec{x}, (1, -1, 1, -1) > \}$.

a. Find a basis for V which consists of mutually perpendicular vectors.

b. Find the distance from the vector $(0, 1, 1, 2)$ to the subspace V.

(10) 5. Define and give an example (no example, no credit) of each of the following terms:

a. Linearly independent set of vectors.

b. LU factorization of a matrix.