(20) 1. Define and give examples of each of the following. No example, no credit.

a. Eigenvector.

b. Linear transformation.

c. Orthogonal basis.

d. Coordinates of a vector.

(15) 2. Let A be a 2×2 matrix with real entries. Suppose R^2 has a basis of eigenvectors $\{(-1, 2),\ (3, 7)\}$, with the corresponding eigenvalues: -4 and $1/2$ respectively.

a. $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = ?$

b. A is similar to a diagonal matrix D. That is $A = PDP^{-1}$.

$D =$ $P =$

c. $A^{99} \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$

(30) 3. Let V be the vector space of polynomials of degree 3 or less. Define the following inner product on V: $< p, q >= p(1)q(1) + p(2)q(2) + 5p(3)q(3) + p(4)q(4)$.

a. Show that if $< p, p >= 0$, than p is the zero polynomial.

b. What "angle" do the vectors $t^3 - 2t^2$ and $t^2 + t + 1$ make with each other?

c. Find the projection of the polynomial $p(t) = t^2 + 6$ onto the subspace of V spanned by the set $\{t + 1,\ t^3 - t^2\}$.

(25) 4. You have the data points $\{(-3, 1),\ (-1.5, 0),\ (-0.5, 2),\ (0.5, 4),\ (1, 3),\ (2, 4)\}$ and want to find a linear combination of the functions $\{1,\ t,\ e^t,\ e^{2t}\}$ which will pass through the given data points.

a. You soon realize that this is impossible. Why? Note: comparing numbers of equations and variables is not sufficient.

b. Find the linear combination of these functions which best fits the data. Be sure to explain what you do. Just pushing the least square botton on the HP will be worth zero points.

(30) 5. Find the matrix representation with respect to the standard basis of the linear transformation which rotates R^3 35 degrees about the line which passes through the origin and the point with coordinates $(1, -2, 3)$. The rotation will be counterclockwise when the origin is viewed from the point $(1, -2, 3)$. Also determine where the point $(1, 1, -2)$ will be sent by the transformation.

(20) 6. The set $S = \{\vec{x}_1,\ \vec{x}_2,\ \vec{x}_3\}$ is linearly independent. Let $T = \{2\vec{x}_1 + \vec{x}_2,\ \vec{x}_2,\ \vec{x}_3\}$.

a. Show that the sets T and S have the same span.

b. Show that the set T is also linearly independent.