Review Sheet for Final Exam
Math 311
Dec. 9, 1996

1. Be sure you understand the following definitions, algorithms, and notations:
   (a) $C[0,1]$, P_n, $M_{m,n}$, R^n, C^n.
   (b) Vector space, subspace, row space, null space, column space, rank, domain, range, image.
   (c) Norm of a vector, dot product, projection, orthonormal basis, Gram-Schmidt procedure.
   (d) Linear independent set, linear dependent set, spanning set, basis, dimension.
   (e) Coordinates, change of basis matrix, rank, eigenvalue, eigenvector.
   (f) Least squares types of problems.
   (g) Polar, cylindrical, and spherical coordinate systems.
   (h) Line integrals, surface integrals.
   (i) Stokes theorem and the divergence theorem.
   (j) Local extremum, Hessian matrix, Lagrange multipliers.
   (k) Grad(f), curl(F), and div(F).
2. You need to be able to do the following:
   (a) Solve a system of linear equations.
   (b) Construct bases for the following subspaces: null space, row space, column space, range of a linear transformation.
   (c) Diagonalize a matrix.
   (d) Given the velocity and/or acceleration vector of some object determine the actual path of the object.
   (e) Given the path of an object determine the unit tangent and normal vectors associated with that path.
   (f) Locate zeros of vector valued functions.
   (g) Locate extremums, both global and constrained, of real valued functions.
   (h) Calculation of work and flux due to a force field.

3. Sample problems:

(a) Let $A = \begin{bmatrix} 1 & -2 & 3 & 8 \\ 6 & 10 & 1 & 0 \\ 11 & 24 & 18 & 10 \\ -2 & 4 & -6 & -16 \\ 3 & -6 & 9 & 24 \end{bmatrix}$. Find a basis for the null space of A. Is the vector $(1, 2, -4, 5)$ in the null space. Does the equation $A\vec{x} = (3, 3, 3, 3, , 3)$ have a solution. Find a basis for the row space of A, the column space of A. What is the rank of A?

(b) Let $A = \begin{bmatrix} 1 & -2 & 3 & 8 \\ -2 & 10 & 1 & 0 \\ 3 & 1 & 18 & 10 \\ 8 & 0 & 10 & 2 \end{bmatrix}$. What are the eigenvalues and eigenvectors of the matrix A. Compute: A^{10}, $\lim_{n\to\infty} A^n$, e^A, $A^{1/2}$.

(c) Find a basis for the subspace $V = \{ p \in P_3 \, : \, p(-1) = p(0), \; p(1) = p(2) \}$.

(d) Find the shortest distance from the point $(2, 3, 1, 5)$ to the subspace $V = \{ \vec{x} \in R^4 \, : \, [\vec{x}, (1, 1, 1, 1)] = 0, [\vec{x}, (-1, 2, 0, 3)] = 0 \}$. The notation $[\cdot, \cdot]$ denotes the standard inner product in R^4.

(e) Find the linear combination of the functions $\{ t^2, \; e^{-t}, \; \sin t, \; \sin 2t, \; 1 \}$, which best fits the data $\{ [-1, 2], \; [-.5, 3], \; [0, 4], \; [1, -5], \; [2, -10], \; [3, -20] \}$.

(f) Let $f(x_1, x_2, x_3) = x_1^2 + \cos(x_1 x_2) + 3x_2^2 + x_3^2$. Let $g(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$.

i. Find all local and absolute extremum of the function f.

ii. Find all extremum of f subject to the condition $g \le 1$.

iii. Let , (t) be a curve in R^3 such that $,'(t) = -\nabla f$. Suppose that , $(0) = (1, -1, 1)$. Calculate , (1) and determine $\lim_{t\to\infty}$, (t).

(g) Let $F(x_1, x_2) = (x_1 - 3x_2^2, x_1^3 + 2x_2 + x_1 + 5x_1 x_2)$. Find those values of $\vec{x}$ such that $F(\vec{x}) = \vec{0}$.

(h) Let $F(x, y, z,) = \left(x^2 - y^2, \, 2e^{xz}, \, \cos xy \right)$ Let $\Omega = \{ (x, y, z) \, : \, |x|, \, |y|, \, |z| \, \le 1 \}$.

i. $\int\int\int_\Omega div(F) \, dV =.$

ii. $\int\int_{\partial\Omega} curl F \cdot \eta \, dS =.$

iii. $\int\int_{\partial\Omega} F \cdot \eta \, dS =.$

(i) Let $x = u^2 - v^2,\ y = 2uv,\ 1 \le u \le 2,\ 1 \le v \le 2$. This rectangle in u-v space will be referred to as R. Denote this transformation by T. That is, $T(u, v) = (u^2 - v^2, 2uv)$.

   i. Graph $T(R)$ in the x-y plane.
   ii. Calculate the area of $T(R)$.
   iii. Show that this is an orthogonal transformation. That is, first find the vectors $\vec{e}_u$ and $\vec{e}_v$, which are the unit vectors pointing in the direction of increasing u and v respectively, and then show that these two vectors are perpendicular to each other at every point in $T(R)$..
   iv. Find the coordinates of the gradient of any function, f, with respect to the vectors $\vec{e}_u$ $\vec{e}_v$. Be sure that all derivatives are in terms of the u and v variables.