

1. (20) A broker offers you a \$10,000 face value Treasury bill with 91 days to maturity at a discount yield of 6.5%.
- (a) What is the price of the bill?

The formulas for the discount yield and bond equivalent yield are  $y_d = \frac{F - P}{F} \frac{360}{t}$  and  $y_b = \frac{F - P}{P} \frac{365}{t}$  respectively. Thus we have

$$\begin{aligned} \frac{6.5}{100} &= \frac{10,000 - P}{10,000} \frac{360}{91} \\ P &= 9,835.69 \end{aligned}$$

- (b) What is the bond equivalent yield of the bill?

$$\begin{aligned} y_b &= \frac{F - P}{P} \frac{365}{t} = \frac{10,000 - 9835.69}{9835.69} \frac{365}{91} \\ &\approx 6.70 \times 10^{-2} \end{aligned}$$

Thus, the bond equivalent yield is approximately 6.7%.

2. (20) Suppose you have two investment options:
- A. Invest \$1000.00 for one year at 6.6%, compounded quarterly.
- B. Invest \$1000.00 for one year at 7% simple interest.

- (a) What are the values of each investment after one year?

Investment A. is worth  $1000 \left(1 + \frac{6.6}{400}\right)^4 \approx \$1067.65$ , and investment B is worth  $1000 \left(1 + \frac{7}{100}\right) = \$1,070.00$ .

- (b) What does the interest rate in investment A. have to equal in order for the two investments to have the same value after one year?

If  $r$  denotes the unknown interest rate, then

$$\left(1 + \frac{r}{400}\right)^4 = 1.07$$

The positive solution to this equation is  $r \approx 6.823\%$ .

3. (50) Let  $n$ , the compounding frequency, equal two. The table below gives the time  $t_0$  term structure of interest rates for two years. Use those interest rates to answer the various parts of this question.

$k$	1	2	3	4
$r(0; k)$	6	6.5	7	7.1

Consider a 10,000 dollar face value default free two year Treasury note, which has a coupon rate of 6.62%.

- (a) How much money is a single coupon payment?

$$\text{payment} = \frac{1}{2} \left( \frac{6.62}{100} \right) 10,000 = 331.00.$$

- (b) What is the present value of the third coupon payment?

$$\begin{aligned} \text{present value} &= \left( 1 + \frac{7}{200} \right)^{-3} 331 \\ &\approx \$298.54. \end{aligned}$$

- (c) What is the time  $t_0$  arbitrage free price of the T-note?

$$\begin{aligned} P &= 331 \left( \left( 1 + \frac{6}{200} \right)^{-1} + \left( 1 + \frac{6.5}{200} \right)^{-2} + \left( 1 + \frac{7}{200} \right)^{-3} + \left( 1 + \frac{7.1}{200} \right)^{-4} \right) \\ &\quad + 10,000 \left( 1 + \frac{7.1}{200} \right)^{-4} \\ &\approx 1218.283 + 8697.603 \approx \$9915.89 \end{aligned}$$

- (d) What is the yield to maturity of the T-note?

The yield to maturity is that value of  $y$  which satisfies the equation

$$\begin{aligned} 9915.89 &= 331 \left( \left( 1 + \frac{y}{200} \right)^{-1} + \left( 1 + \frac{y}{200} \right)^{-2} + \left( 1 + \frac{y}{200} \right)^{-3} + \left( 1 + \frac{y}{200} \right)^{-4} \right) \\ &\quad + 10,000 \left( 1 + \frac{y}{200} \right)^{-4}. \end{aligned}$$

The solution to this equation is  $y \approx 7.078\%$ . Using the bond calculator, the following values are entered in the given stacks.

99.1589 in PV, 6.662 in PMT, then put two dates two years apart on the stack. For example 1.011990 and 1.011992 are valid. Then push “YTM” or “f 1/x”.

- (e) Suppose you can buy this bond for \$9500. Explain how you can turn this opportunity into an arbitrage. Be specific and compute the amount of money that can be earned which will be risk free.

Buy the bond for \$9500 and then sell four CD's with face values of 331, 331, 331, and 10,331 and maturing in 6, 12, 18, and 24 months respectively. The amount of money received for the CD's is their present value which equals 9915.89. Thus, you have a net cash worth of  $9915.98 - 9500 = 415.89$ . Your payment obligations for the CD's will be taken care of by the bond payments you receive.

In addition to the 415.89 cash difference you can also realize interest on this money for 2 years at 7.1%, which brings the grand total of arbitrage money to

$$415.89 \left( 1 + \frac{7.1}{200} \right)^4 \approx \$478.16$$

4. (10) Suppose you borrow \$12,000 from a bank at  $7\frac{5}{8}\%$  interest compounded monthly.

- (a) If your monthly payments are \$250, how long will it take you to pay off the loan, and what is the loan balance immediately after the next to the last payment?

Using the bond calculator enter 12,000 in PV, -250 in PMT  $\frac{7.625}{12}$  in i and 0 in FV, then push n. The calculator has as output the number 58. However, the calculator rounds to the next highest interger. So the correct answer is 57 plus a 58<sup>th</sup> payment of  $110.08 \left( 1 + \frac{7.625}{1200} \right) \approx 110.78$ . The value 110.08 is the amount left after the 57<sup>th</sup> payment.

- (b) You decide to pay off the loan with the following sequence of payments.

month	1	2	6
payment	1500	2500	X

The payments being made at the end of the indicated month. For example, a \$2500 payment is made at the end of the second month. Determine the size, X, of the last payment.

Let  $B_i$  denote the balance after the  $i^{\text{th}}$  payment. The final payment will be  $B_2$  plus the interest it earns for 4 months. Thus,

$$\begin{aligned} B_1 &= \left(1 + \frac{7.625}{1200}\right) 12000 - 1500 \approx 10576.25 \\ B_2 &= \left(1 + \frac{7.625}{1200}\right) 10576.25 - 2500 \approx 8143.45 \\ X &= \left(1 + \frac{7.625}{1200}\right)^4 8143.45 \approx 8352.41 \end{aligned}$$