1. (15) A U.S. government securities dealer provided the following quotes on 10/16/00

| Coupon Rate | Maturity | Bid | Asked |
|---|---|---|---|
| 6.75% | 5/15/04 | 100:24 | 100:28 |

A customer purchased $100,000 face value of the security, with settlement on 10/17/00. Determine the invoice price to the penny.

The invoice price equals the flat price plus any accrued interest. Thus,

$$\begin{aligned} \text{invoice price} &= 100,000\left(\frac{100+\frac{28}{32}}{100}\right)+100,000\left(\frac{6.75}{200}\right)\left(\frac{155}{184}\right) \\ &= 100,875.00+2,843.07 \\ &= 103,718.07 \end{aligned}$$

2. (15) Find the approximate percentage price change (to 3 decimal places) in the price of a T-note with a duration of 11 half years, if its yield to maturity decreases from 6.5% to 6.25%.

The percentage change in price is equal to

$$\begin{aligned} \frac{P(y+\triangle)-P(y)}{P(y)} &\approx -\frac{D}{200+y}\triangle \\ &= -\frac{11}{206.5}(-0.25) \\ &\approx 0.0133\,. \end{aligned}$$

Thus, the percentage change in price is 1.33.

3. (35) Let $B_1$ denote a \$100 face value zero coupon bond with yield to maturity 6.5% which matures in 5 years, and let $B_2$ denote a \$100 face value zero coupon bond with yield to maturity 7.25% which matures in 8 years. A portfolio consists of 100 units of bond $B_1$ and $x$ units of bond $B_2$.

   (a) What are the Macaulay durations and convexities of each of these bonds?

   The Macaulay duration of any zero coupon bond equals the number of half years to maturity. The convexity of a zero coupon bond equals $\frac{M\,(M+1)}{(200+y)^2}$. Thus, for these two bonds we have

$$\begin{aligned} D_1 &= 10\,; \quad C_1 = \frac{10 \times 11}{(206.5)^2} \approx 0.00258 \\ D_2 &= 16\,; \quad C_2 = \frac{16 \times 17}{(207.25)^2} \approx 0.00633 \end{aligned}$$

   (b) What is the value of $x$ for this portfolio to be delta hedged?

   Set the duration of the portfolio equal to zero and solve of $x$.

$$100P_1\,(MD_1) + xP_2\,(MD_2) = 0\,,$$

   where $(MD_i)$ denotes the modified duration of bond $B_i$. Thus, we have

$$\begin{aligned} x &= -100\frac{P_1\,(MD_1)}{P_2\,(MD_2)} \\ &\approx -100\left(\frac{72.627}{56.567}\right)\left(\frac{0.048426}{0.077201}\right) \\ &\approx -80.536 \end{aligned}$$

   (c) If $x = -50$, what is the value of the portfolio at time $t_0$?

   The value of the portfolio equals

$$\begin{aligned} \Pi_0 &= 100 \times 72.627 - 50 \times 56.567 \\ &= 4,434.35 \end{aligned}$$

(d) If $x = -50$, what is the convexity of the portfolio at time $t_0$?

The convexity equals

$$\begin{aligned} C &= \frac{x_1 P_1}{\Pi_0} C_1 + \frac{x_2 P_2}{\Pi_0} C_2 \\ &= \frac{100 \times 72.627}{4,434.35}(0.00258) - \frac{50 \times 56.567}{4,434.35}(0.00633) \\ &\approx 0.000188 \end{aligned}$$

4. (35) The time $t_0$ modified durations and convexities of two bonds and other particulars of theirs are in the table below. Each of the bonds has a face value of \$100.

| | Years | $R$ (%) | $y$ (%) | $P_0$ | (MD) | Convexity |
|---|---|---|---|---|---|---|
| $B_1$ | 8 | 6 | 6.75 | 95.422 | 0.0622 | 0.00472 |
| $B_2$ | 4 | 5.825 | 6.25 | 98.516 | 0.0350 | 0.00148 |

(a) Suppose there is a uniform shift in the yields to maturity equal to $\triangle = -0.05$ immediately after the bonds are purchased at time $t_0$. Compute the price of bond $B_2$ 1.5 years later.

Use the bond calculator, the price of bond $B_2$ with a yield of 6.2 and 2.5 years to maturity is approximately 99.144

(b) Assume a reinvestment rate of 5.775%. Under these assumptions, what is the total return to time $t_j$ of bond $B_2$ if it is sold after 1.5 years; what is the total yield of bond $B_2$ if it is sold after 1.5 years?

The total return equals

$$\begin{aligned} (TRTM)_3 &= \frac{5.825}{2}\left(1+\frac{5.775}{200}\right)^2 + \frac{5.825}{2}\left(1+\frac{5.775}{200}\right) \\ &\quad + \frac{5.825}{2} + 99.144 - 98.516 \\ &\approx 9.62 \end{aligned}$$

The total yield of this bond after it is sold is that value of $y$ such that

$$\begin{aligned} \text{price paid} &= \left(1+\frac{y}{200}\right)^{-3} (\text{value of investment}) \\ 98.516 &= \left(1+\frac{y}{200}\right)^{-3} (9.62 + 98.516) \end{aligned}$$

The solution of this equation is

$$y \approx 6.3$$

(c) A portfolio consists of 100 units of $B_1$ and 150 units of $B_2$. What is the time $t_0$ duration of this portfolio?

The value of the portfolio is first calculated

$$\begin{aligned}\Pi_0 &= 100 \times 95.422 + 150 \times 98.516 \\ &= 24,319.60\end{aligned}$$

The duration equals

$$\begin{aligned}D &= \frac{x_1 P_1}{\Pi_0}(MD_1) + \frac{x_2 P_2}{\Pi_0}(MD_2) \\ &= \frac{100 \times 95.422}{24,319.60}(0.0622) + \frac{150 \times 98.516}{24,319.60}(0.035) \\ &\approx 0.0458\end{aligned}$$