

1. (40) A U.S. Treasury bond matures on November 15, 2001. The bond is purchased with settlement on December 16, 2000. The basic data for this bond on the settlement date is shown in the table below.

Coupon Rate, R	Face Value, F	Invoice Price, P
6.5%	\$50,000	\$49,750

- (a) Calculate that part of the invoice price which is the accrued interest. The coupon period in which settlement occurs has 181 days. There are 31 days from the last coupon payment to the settlement date. Thus

$$\text{accrued interest} = 50,000 \frac{6.5}{200} \frac{31}{181} = 278.32$$

- (b) Calculate the yield to maturity of the bond.

The flat price of the bond equals $49,750 - 278.32 = 49471.68$. For a \$100 face value bond this is equivalent to $\frac{49471.68}{500} \approx 98.94336$. Entering this price and other data into the bond calculator produces a yield of 7.7089%.

- (c) Calculate the bond's Macaulay duration.

$$\begin{aligned} \omega_1 &= \frac{RF}{200P} \left(1 + \frac{y}{200}\right)^{-150/181} \\ &= \frac{6.5 \times 50000}{200 \times 49750} \left(1 + \frac{7.7089}{200}\right)^{-150/181} \\ &\approx 0.0316 \\ \omega_2 &= \frac{1}{P} \left(\frac{RF}{200} + F\right) \left(1 + \frac{y}{200}\right)^{-(1+150/181)} \\ &= \frac{1}{49750} \left(\frac{6.5 \times 50000}{200} + 50000\right) \left(1 + \frac{7.7089}{200}\right)^{-(1+150/181)} \\ &\approx 0.9683 \end{aligned}$$

Thus, the Macaulay duration equals

$$\begin{aligned} D &= \frac{150}{181} \omega_1 + \left(1 + \frac{150}{181}\right) \omega_2 \\ &\approx 1.797 \end{aligned}$$

- (d) Assume there is a change in the yield of -0.03% immediately after the bond is purchased. Use the Macaulay duration to estimate the percentage change in the invoice price.

$$\begin{aligned} \frac{P(y + \Delta) - P(y)}{P(y)} &\approx -\frac{D}{200 + y} \Delta \\ &\approx -\frac{1.797}{207.71} (-0.03) \\ &\approx 0.000259 \end{aligned}$$

Thus, the percentage price change is approximately 0.026% .

- (e) Suppose the bond is purchased on June 15, 2001 with a yield to maturity of 7% . What would the invoice price of the bond be?

The coupon period in which the settlement date occurs is the very last payment period. This period has 184 days in it and there are 153 days from settlement to the last payment. Thus, the price equals

$$\begin{aligned} P &= \left(1 + \frac{7}{200} \frac{153}{184}\right)^{-1} \left(1 + \frac{6.5}{200}\right) 50,000 \\ &\approx 50,165.03 \end{aligned}$$

2. (10) You have a liability of $\$50,000$ due in 1.5 years. Construct a synthetic zero coupon bond from the following two bonds which will satisfy the liability. Both bonds mature in 1.5 years, and have face values of $\$100$.

Bond	R	y
B_1	6.25	6.5
B_2	6.75	6.875

Let x_1 and x_2 denote the number of units of bonds B_1 and B_2 used to construct the synthetic zero coupon bond. Then we must have

$$\begin{aligned} \left(\frac{6.5}{200}100\right)x_1 + \left(\frac{6.75}{200}100\right)x_2 &= 0 \\ (x_1 + x_2)100 &= 50,000 \end{aligned}$$

The solution to this system is

$$\begin{aligned} x_1 &= 6750 \\ x_2 &= -6250 \end{aligned}$$

3. (10) The Macaulay duration of a zero coupon bond equals M , the number of half years left to maturity of the bond, if the bond is bought on a coupon date. Find the Macaulay duration for a zero coupon bond when the settlement date falls between two coupon payment and $M > 1$.

The duration equals $D = \sum_{k=1}^M \left(k - 1 + \frac{z}{x}\right) \omega_k$. For a zero coupon bond $R = 0$. Hence all of the ω_k 's equal zero except for ω_M .

$$\omega_M = \frac{F}{P} \left(1 + \frac{y}{200}\right)^{-(M-1+z/x)} = 1.$$

Thus,

$$D = \left(M - 1 + \frac{z}{x}\right) \omega_M = \left(M - 1 + \frac{z}{x}\right).$$

4. (30) The table below gives data for some U.S. Treasury securities. All three bonds have a face value of \$100. The times to maturity are in years.

Issue	Maturity	Coupon	Price	Yield
B_1	0.5	0	96.85	6.5
B_2	1	5.5	99.165	6.375
B_3	1.5	5.625	99.12	6.25

- (a) Use linear interpolation to fill in the missing yield. Then use your bond calculator to determine the missing price.

The boldfaced values in the above table are the filled in answers.

- (b) Using the data from the previous problem determine the first two terms of the time t_0 term structure of interest rates $\{r(0, k)\}_{k=1}^3$. That is determine $r(0; 1)$ and $r(0; 2)$.

$$r(0; 1) = 6.5$$

The formula which is used to determine $r(0; 2)$ is

$$99.165 = \frac{5.5}{2} \left(1 + \frac{6.5}{200}\right)^{-1} + \left(\frac{5.5}{2} + 100\right) \left(1 + \frac{r(0; 2)}{200}\right)^{-2}$$

The positive solution to this equation is

$$r(0; 2) \approx 6.373.$$

5. (10) A newspaper reported the following data on some \$100 face value bonds. The paper got the coupon rates and yields correct, but made some mistakes in the prices. Determine which of the reported prices have to be incorrect and why.

Bond	Price	Coupon Rate	Yield
B_1	90	6	9
B_2	96	9	8
B_3	110	8	6
B_4	105	0	5
B_5	107	7	9
B_6	100	6	6

There are two facts needed to solve this puzzle.

- (a) If the yield and coupon rate are equal, then the price and face value are also equal.
- (b) If the yield increases, then the price decreases, and conversely.

Thus, the prices for bonds B_2 , B_4 , and B_5 must be incorrect.