1. (20) A broker offers you a \$1,000 face value Treasury bill with 183 days to maturity at a bond equivalent yield of 6.0%.

   (a) What is the price of the bill?

$$
\begin{aligned}
y_b &= \frac{F-P}{P}\frac{365}{t} \\
\frac{6}{100} &= \frac{1000-P}{P}\frac{365}{183} \\
P &= 970.80
\end{aligned}
$$

   (b) What is the discount yield of the bill?

$$
\begin{aligned}
y_d &= \frac{F-P}{F}\frac{360}{t} \\
&= \frac{1000-970.80}{1000}\frac{360}{183} \\
&\approx 5.7\times 10^{-2}
\end{aligned}
$$

   Or $y_d = 5.7\,\%$.

The table below contains a time t_0 interest rate structure with compounding frequency $n = 2$. Use this table for problems 2 and 3 below.

$r(0,1)$	$r(0,2)$	$r(0,3)$	$r(0,4)$	$r(0,5)$	$r(0,6)$
5.0	5.0	5.25	5.5	6	6.125

2. (20) Explain how to construct a synthetic forward $(\mathrm{CD})^{(2)}_{4,6}$. Determine its effective interest rate.

Sell $(\mathrm{CD})^{(2)}_{0,4}$ and buy $(\mathrm{CD})^{(2)}_{0,6}$ with the monies received from selling the first CD. The effective rate is r, where r satisfies the equation

$$
\left(1+\frac{r}{200}\right)^2 = \left(1+\frac{6.125}{200}\right)^6\left(1+\frac{5.5}{200}\right)^{-4}
$$

The solution to this equation is $r = 7.38\,\%$.

3. (40) Consider a 1,000 dollar face value default free two year Treasury note, which has a coupon rate of 6.0%.

(a) How much money is a single coupon payment?

$$\begin{aligned} \text{payment} &= \frac{R}{200}F \\ &= \frac{6}{200}1000 = 30.00 \end{aligned}$$

(b) What is the present value of the third coupon payment?

$$\begin{aligned} \text{PV} &= \left(1+\frac{5.25}{200}\right)^{-3} 30 \\ &= 27.76 \end{aligned}$$

(c) What is the time t_0 arbitrage free price of the T-note?

$$\begin{aligned} P &= \left(1+\frac{5}{200}\right)^{-1} 30 + \left(1+\frac{5}{200}\right)^{-2} 30 + \left(1+\frac{5.25}{200}\right)^{-3} 30 + \left(1+\frac{5.5}{200}\right)^{-4} 1030 \\ &= 1009.66 \end{aligned}$$

(d) What is the yield to maturity of the T-note?

The yield to maturity satisfies the equation

$$1009.66 = \left(1+\frac{y}{200}\right)^{-1} 30 + \left(1+\frac{y}{200}\right)^{-2} 30 + \left(1+\frac{y}{200}\right)^{-3} 30 + \left(1+\frac{y}{200}\right)^{-4} 1030$$

The solution to this equation is $y = 5.48\,\%$.

(e) Suppose you can buy this T-note for $950. Explain how you can turn this opportunity into an arbitrage. Be specific and compute the amount of money that can be earned which will be risk free.

Buy the bond and at the same time sell four CD's, which mature in 6 months, 12 months, 18 months and two years. The face values of the CD's are 30, 30, 30, and 1030 respectively. The price received for selling these CD's is $1009.66. Since the cost of the bond is only $950 there is an instant profit of $59.66. This profit can be increased by investing it for the two years. The treasury notes coupons are used to pay off the CD's when they mature.

4. (20) Suppose it is possible to buy and sell certificates of deposit at the following interest rates: $r(0,1) = 6\%$, $r(0,2) = 5\%$, and $r(0,3) = 3\%$. This presents an arbitrage opportunity. Explain how you would use this opportunity to make a risk free profit of \$10,000 without using any of your own money.

There is a relation which consecutive interest rates in the term structure of interest rates must satisfy in order that an arbitrage opportunity not be present. The relation is

$$\left(1 + \frac{r(0,k+1)}{100n}\right)^{k+1} > \left(1 + \frac{r(0,k)}{100n}\right)^{k}$$

A quick check of this values for the given interest rate leads to

$$\begin{aligned}\left(1 + \frac{3}{200}\right)^{3} &= 1.0456 \\ \left(1 + \frac{5}{200}\right)^{2} &= 1.050 \\ \left(1 + \frac{6}{200}\right) &= 1.03\end{aligned}$$

Thus, there is an arbitrage opportunity in buying and selling $(\mathrm{CD})^{(2)}_{0,2}$ and $(\mathrm{CD})^{(2)}_{0,3}$. Sell $(\mathrm{CD})^{(2)}_{0,3}$ for a face value of \$1 and buy the amount received worth of $(\mathrm{CD})^{(2)}_{0,2}$. The money received after the first year equals

$$\left(1 + \frac{5}{200}\right)^{2}\left(1 + \frac{3}{200}\right)^{-3} = 1.005$$

Thus, after 1 year you will receive a payment of \$1.005, one dollar of which can be used to pay of the CD which will mature in an addional 6 months. So without even using the interest the 1.005 can earn over a 6 month period there is an arbitrage profit of 0.005. Thus, to earn a \$10,000 profit sell

$$\frac{10000}{0.005} = 2.0 \times 10^{6}$$

dollars worth of the CD which matures in 18 months. The profit earned is at least

$$\left(1 + \frac{5}{200}\right)^{2}\left(1 + \frac{3}{200}\right)^{-3} \times 2.0 \times 10^{6} - 2.0 \times 10^{6} = 9461.08$$

The fact that this is slightly less than 10,000 is due to roundoff error. The 0.005 is a rounded off value of 0.0047.....