

Bonds	Coupon Rate	Par Value	Time to Maturity
A	0.04	1000	3
B	0.05	100	3
C	0	1000	3

t	0.5	1	1.5	2	2.5	3
$\delta(t)$	0.99	0.98	0.95	0.9	0.85	0.8
$\sigma(t)$	0.99	1.97	2.92	3.82	4.67	5.47

- 1a. The present value is calculated by computing the present values of the cash flows.

Present Value

Bond A	909.40
Bond B	93.68
Bond C	800.00

- 1b. Let  $m$  and  $n$  denote the number of B and C bonds respectively. The following equations arise from equating the coupon payments and par values of the portfolio of bonds B and C with the coupon payments and par value of bond A.

$$(0.05/2) \cdot 100m = (0.04/2) \cdot 1000$$

$$100m + 1000n = 1000$$

The solution to this system of equations is  $m = 8$  and  $n = 1/5$ .

- 1c. The delivery price is going related to the present value of the 10 bonds by the formula

$F + \delta(T)D = A$  where  $F$  is the value of the contract, which we set equal to zero, and  $T$  is the time to delivery, which in this case is 2 year. However, at the time of delivery Bond A has one coupon payment before maturity plus of course payment at maturity.

Thus, the present value of one A equals 8330

$$\text{Thus, } D = A / \delta(2) = 9255.56$$

2. A fair price means that both parties to the transaction will have the same amount of money after the bond matures.

If the owner of the bond receives \$950, and invests it at 8% compounded quarterly, then after 1 and 1/2 years he will have  $950 \cdot (1 + 0.08/4)^6 = 1069.85$  dollars

The owner of the bond on the other hand will invest the first coupon payments at 8% for one year, a second coupon payment for 1/2 year and then receive the final coupon payment and the par value after one and 1/2 years. This comes to a grand total of 1062.46

So someone buying this bond is paying \$7.39 too much, and \$950 is not a fair price.

3. The formula that relates present value, delivery price, and contract price is  $F + \delta(T)D = A$ , where  $T$  is the number of years to settlement,  $A$  is the current price of the asset, and  $D$  is the delivery price. In this problem we know the following:

$F = \$5,000$ ,  $T = 1$ , and  $D = \$20,000$ . Using the discount factors above we have

$$A = 24600$$

The value of the corn one year from now is its current value divided by  $\delta(1)$ . So we get

$$A(1) = 25102.04 \text{ Dividing this by } 10,000 \text{ we see that a bushel of corn costs.}$$