

The data in the table below is for all of the problems below.

$t$	1/2	1	3/2
$\delta(t)$	0.99	0.96	0.95
$r(t)$	0.0202	0.050	0.024
$c(t)$	150.00	200.00	50.00
$s(t)$	<b>0.0202</b>	<b>0.04124</b>	<b>0.03449</b>
$f(t)$	<b>0.0202</b>	<b>0.0625</b>	<b>0.02105</b>

$\delta(t)$  is the discount factor,  $r(t)$  is the 6 month spot rate in effect at  $t - 1/2$ ,  $s(t)$  is the  $t$ -year spot rate,  $f(t)$  is the 6 month forward rate at  $t - 1/2$ , and  $c(t)$  is a cash flow.

1. (6) Fill in the table.

$$s(1/2) = 2\left(\frac{1}{\delta(1/2)} - 1\right) = 0.0202, s(1) = 2\left(\sqrt{\frac{1}{\delta(1)}} - 1\right) = 0.04124$$

$$s(3/2) = 2\left(\left(\frac{1}{\delta(3/2)}\right)^{1/3} - 1\right) = 0.03449, f(1/2) = s(1/2)$$

$$f(1) = 2\left(\frac{\delta(1/2)}{\delta(1)} - 1\right) = 0.0625, f(3/2) = 2\left(\frac{\delta(1)}{\delta(3/2)} - 1\right) = 0.02105$$

2. (4) In addition to the cash flow  $c(t)$  given in the table you also receive \$100.00 at  $t = 0$ . Find the present value of this cash flow.

$$\begin{aligned} \text{present value} &= V = 100 + 150\delta(1/2) + 200\delta(3/2) + 50\delta(3/2) \\ &= 488.00 \end{aligned}$$

3. (5) Find the total return of the cash flow including  $c(0)$  in your calculations.

$$\begin{aligned} \text{total return} &= \text{TR} = 100\left(1 + \frac{s(1/2)}{2}\right)\left(1 + \frac{r(1)}{2}\right)\left(1 + \frac{r(3/2)}{2}\right) \\ &\quad + 150\left(1 + \frac{r(1)}{2}\right)\left(1 + \frac{r(3/2)}{2}\right) + 200\left(1 + \frac{r(3/2)}{2}\right) + 50 \\ &= 104.777673 + 155.595 + 202.40 + 50 \\ &= 512.77 \end{aligned}$$

4. (5) If  $V$  is the present value of the cash flow does  $\frac{V}{\delta(3/2)}$  equal the total return. Explain.

$$\frac{V}{\delta(3/2)} = \frac{488.00}{0.95} = 513.68.$$

The expression  $\frac{V}{\delta(3/2)}$  will equal the total return if the 6 month spot rates,  $r(t)$ , in effect in the future equal the 6 month forward rates,  $f(t)$ , and in this case they do not.