Math 325 - 500 Solutions Quiz 6 Oct. 23, 2009

1. (15) Assume it's Aug. 15, 2003, and use the discount factors in table one for this problem. Bond A is a 5 year, par \$1000 coupon bond with an 8% coupon rate.

a. What is the price of this bond?

From table 1 we have $\delta(5) = 0.8434$ and $\sigma(5) = 9.2956$. Thus, the price V equals

$$\begin{aligned} V &= \delta(5)1000 + \sigma(5)40 \\ &= 1215.224 \end{aligned}$$

b. What is its yield to maturity?

The yield to maturity satisfies the equation

$$1215.224 = 1000\left(1 + \frac{y}{2}\right)^{-10} + 1000\left(\frac{0.08}{y}\right)\left(1 - \left(1 + \frac{y}{2}\right)^{-10}\right)$$

It's solution, I used Maple and Excel, is

$$y = 0.032958$$

c. Assuming reinvestment at the 6 month forward rates, what is the bond's total return?

$$\text{Total return} = \frac{V}{\delta(5)} = \frac{1215.224}{0.8434} = 1440.86$$

d. Assuming reinvestment at its yield to maturity, what is the bond's total return?

$$\text{Total return} = V\left(1 + \frac{y}{2}\right)^{10} = 1215.224\left(1 + \frac{0.032958}{2}\right)^{10} = 1431.00$$

e. If you could sell the bond right now for its par value would you do so. Be sure to explain.

Since the yield is less than the coupon rate, which indicates that current interest rates are less than the coupon rate, I would not sell my bond for its par value; 10% is better than 3.29%.

2. (15) You decide to buy a home, and your bank will lend you the \$150,000 needed at 5.5% compounded monthly.

The basic formula used in this problem is

$$B(i) = V\left(1 + \frac{0.055}{12}\right)^i - L\left(\frac{12}{0.055}\right)\left(\left(1 + \frac{0.055}{12}\right)^i - 1\right).$$

$B(i)$ gives the balance owed on a loan of V dollars at 5.5% interest per year compounded monthly, immediately after the i^{th} payment of L dollars.

a. What must your monthly payment equal in order to pay off the loan in 25 years?

The appropriate equation is

$$0 = 150000\left(1 + \frac{0.055}{12}\right)^{25*12} - L\left(\frac{12}{0.055}\right)\left(\left(1 + \frac{0.055}{12}\right)^{25*12} - 1\right),$$

and its solution is

$$p_1 = \frac{150000\left(1 + \frac{0.055}{12}\right)^{25*12}}{\left(\frac{12}{0.055}\right)\left(\left(1 + \frac{0.055}{12}\right)^{25*12} - 1\right)} = 921.1312384\,.$$

I'm calling it p_1 as this value will be used throughout the rest of this problem.

b. Suppose after 1 year (12 payments) you increase your monthly payment by \$100. How long before the loan is paid off?

We first need to find the balance of our loan after 12 payments, which leads to the equation

$$B(12) = V_2 = 150000\left(1 + \frac{0.055}{12}\right)^{12} - 921.1312384 * \left(\frac{12}{0.055}\right)\left(\left(1 + \frac{0.055}{12}\right)^{12} - 1\right) = 147,124.66\,.$$

With this as our new balance we have the equation

$$0 = 147,124.66\left(1 + \frac{0.055}{12}\right)^i - (p_1 + 100)\left(\frac{12}{0.055}\right)\left(\left(1 + \frac{0.055}{12}\right)^i - 1\right),$$

and its solution is

$$t_2 = 236.1518187 \text{ months.}$$

Thus, the total time in years before the loan is paid off is

$$\text{total time} = 1 + \frac{236.1518187}{12} = 20.67931823 \text{ years} = 20 \text{ years and } 8.1518 \text{ months.}$$

c. Suppose after another 2 years (36 payments in total) you decide to increase your monthly payment by an additional $200. How long before the loan is paid off, and what will your last payment equal?

As in part b. we need to determine the remaining balance, which is

$$\begin{aligned} V_3 &= B(24) \\ &= 147,124.66\left(1+\frac{0.055}{12}\right)^{24} - (921.1312384+100)*\left(\frac{12}{0.055}\right)\left(\left(1+\frac{0.055}{12}\right)^{24}-1\right) \\ &= 138,347.40 \end{aligned}$$

The time to pay off the loan is found by solving the equation

$$0 = 138,347.40\left(1+\frac{0.055}{12}\right)^{i} - (921.1312384+300)*\left(\frac{12}{0.055}\right)\left(\left(1+\frac{0.055}{12}\right)^{i}-1\right)$$

The solution is

$$i = 160.1714155 .$$

Thus, we need another 160 months plus a bit to pay off the loan. The total time paying off the loan in years is

$$\begin{aligned} 3+\frac{160.1714155}{12} &= 16.34761796 \text{ months} \\ &= 16 \text{ years and } 4.17141552 \text{ months.} \end{aligned}$$

The balance owed after the penultimate payment is

$$\begin{aligned} \text{balance} &= 138,347.40\left(1+\frac{0.055}{12}\right)^{160} - (921.1312384+300)*\left(\frac{12}{0.055}\right)\left(\left(1+\frac{0.055}{12}\right)^{160}-1\right) \\ &= 208.75 . \end{aligned}$$

The final payment is

$$208.75\left(1+\frac{0.055}{12}\right) = 209.71$$

d. How much interest will you save if you pay the note off as in c. versus paying it off in 25 years?

The amount of interest paid if we take 25 years is

$$\begin{aligned} \text{interest}_1 &= 25*12*921.1312384 - 150000 \\ &= 126,339.37 \end{aligned}$$

The amount of interest paid using different monthly payments is

$$\begin{aligned} \text{interest}_2 &= 12p_1 + 24(p_1+100) + 160(p_1+300) + 209.71 - 150000 \\ &= 81,151.43 \end{aligned}$$

Thus, the amount of interest saved is

$$126,339.37 - 81,151.43 = 45,187.94$$