Solutions to Home work Set 1

1. (10) An insurance company receives a premium of 12 million dollars.

   (a) In exchange for the premium the insurance company promises to pay 14 million dollars in 8 years. Assume the insurance company can invest the entire 12 million and earn interest at $r\,\%$ compounded quarterly. What must r equal for the insurance company to meet its obligation?

$$\begin{aligned} 12 \times 10^6 \left(1 + \frac{r}{400}\right)^{32} &= 14 \times 10^6 \\ \left(1 + \frac{r}{400}\right)^{32} &= \frac{14}{12} \\ r &= 400\left(\left(\frac{14}{12}\right)^{1/32} - 1\right) \\ &= 1.932\,\% \end{aligned}$$

   (b) In exchange for the 12 million dollar premium the insurance company promises to pay 500,000 dollars every quarter. If the insurance company can invest the premium at 6.5% compounded quarterly, how many full payments can the insurance company make, and how much money will be left after the last full payment?

   If B_n represents the balance after n payments, we have

$$B_n = 12 \times 10^6 \left(1 + \frac{6.5}{400}\right)^n - 500,000 \frac{\left(1 + \frac{6.5}{400}\right)^n - 1}{6.5/400}$$

   Setting B_n equal to 0 and solving for n we have $n = 30.665$. Thus, the insurance company can make 30 full payments. After the 30$^{\text{th}}$ payment the balance is

$$\begin{aligned} B_{30} &= 12 \times 10^6 \left(1 + \frac{6.5}{400}\right)^{30} - 500,000 \frac{\left(1 + \frac{6.5}{400}\right)^{30} - 1}{6.5/400} \\ &= 327,928.18 \end{aligned}$$

2. (10) A used car is purchased for \$10,000 by financing it as follows: \$2,000 down and the balance paid off monthly at 12% for three years.

(a) What are the monthly payments?

Using the same formula as before, that is

$$B_{36} = 8000\left(1+\frac{12}{1200}\right)^{36} - x\frac{\left(1+\frac{12}{1200}\right)^{36} - 1}{12/1200}$$

Set B_{36} equal to zero and solve for x, which is the monthly payment.

$$\begin{aligned} x &= \frac{8000\left(1+\frac{12}{1200}\right)^{36}}{\left[\frac{\left(1+\frac{12}{1200}\right)^{36} - 1}{12/1200}\right]} \\ &= 265.71 \end{aligned}$$

(b) At the end of three years how much will have been spent?

After the three years the total amount spent is

$$2000 + 36x = 2000 + 36\,(265.71) = 11,565.56$$

(c) If instead of buying the car the \$2000 down payment and the monthly payments are invested at 6.5% per year compounded monthly. How much money will the investment be worth after 3 years?

The investment's worth equals

$$\begin{aligned} A_{36} &= 2000\left(1+\frac{6.5}{1200}\right)^{36} + 265.71\frac{\left(\left(1+\frac{6.5}{1200}\right)^{36} - 1\right)}{6.5/1200} \\ &= 2,429.34 + 265.71\,(39.63) \\ &= 12,959.88 \end{aligned}$$

3. (10) You have two investment options:

Option A pays 4.6% compounded semiannually, and no taxes have to be paid on the income

Option B. pays 6.8% compounded semiannually, but taxes are paid on the income at a marginal rate of 31%.

(a) If you can invest $2000 for three years in one of the two options, which one is better, and how much better is it. That is, how much more money is earned in the better of the two investment options. Be sure to take into account the fact that the second investment has tax liabilities.

Option A after 3 years is worth

$$2000\left(1+\frac{4.6}{200}\right)^6 = 2292.36$$

Option B after 3 years is worth

$$2000\left(0.69\left(1+\frac{6.8}{200}\right)^2+0.31\right)^3 = 2300.18$$

Thus, option B is a better investment.

(b) What must the marginal rate of taxation equal in order for the two investment options to be equivalent?

If r is the marginal tax rate, then an equation, which r must satisfy is

$$2000\left(\left(1-\frac{r}{100}\right)\left(1+\frac{6.8}{200}\right)^2+\frac{r}{100}\right)^3 = 2292.36$$

The solution to this is

$$r = 32.720\,\%$$

4. (10) A company sells a 5 year annuity for \$2000. The annuity will make the following sequence of payments:

Year	1	2	3	4	5
Payment	500	500	500	500	1500

(a) What is the internal rate of return of the annuity?

We want to find that interest rate for which the present values of these payments sum to 2,000. Since nothing has been said to the contrary we'll assume that compounding is done yearly. The equation which the interest rate satisfies is

$$\begin{aligned} 2000 &= 500\left(\left(1+\frac{r}{100}\right)^{-1}+\left(1+\frac{r}{100}\right)^{-2}+\left(1+\frac{r}{100}\right)^{-3}+\left(1+\frac{r}{100}\right)^{-4}\right) \\ &\quad +1500\left(1+\frac{r}{100}\right)^{-5} \end{aligned}$$

The solution to this equation is

$$r = 18.013$$

(b) Suppose interest rates are forecasted to be 5% compounded semi-annually for the 5 years. How much should you be willing to pay for the annuity?

This amounts to computing the present value of the above 5 payments assuming that money is compounded two times per year.

$$\begin{aligned} \text{present value} &= 500\left(\left(1+\frac{5}{200}\right)^{-2}+\left(1+\frac{5}{200}\right)^{-4}+\left(1+\frac{5}{200}\right)^{-6}+\left(1+\frac{5}{200}\right)\right)^{-8} \\ &\quad +1500\left(1+\frac{5}{200}\right)^{-10} \\ &= 2942.20 \end{aligned}$$