Math 325 Solutions to Home work Set 6 Nov. 8, 2001

The solutions to the answers provided below can be found in the text version of a Maple worksheet which is attached to the end of this file.

1. (15) A bond with a face value of \$1000 matures December 21, 2025. Its coupon rate is 6%, and its yield to maturity on November 8, 2001 is 4%. Determine the following.

   (a) Flat price, accrued interest, and invoice price on November 8, 2001.

   $$\text{flat price} = 1307.58, \text{ accrued interest} = 22.95, \text{ invoice price} = 1330.53$$

   (b) Its duration and convexity on November 8, 2001.

   $$\text{duration} = 28.403, \text{ convexity} = 0.0272$$

   (c) Assume that there is an immediate decrease in the yield of 50 basis points. Use your answers in part b. find a second order approximation to the new invoice price of the bond.

   $$\text{new price} = 1427.67$$

2. (30) The table below contains some data on two different bonds, each of which has a face value of \$100. Use this data to answer the questions which follow.

Bond	Matures	R	y
1	Dec. 10, 2002	6	3
2	May 15, 2010	5	4

   Assume a portfolio is constructed by buying some quantities of each of these bonds, and that the settlement date for the purchases is Nov. 15, 2001.

   (a) What is the invoice price of each bond?

   $$\begin{aligned} \text{price for } B_1 &= 105.72 \\ \text{price for } B_2 &= 107.50 \end{aligned}$$

   (b) What is the Macaulay duration of each bond?

   $$D_1 = 2.052, \qquad D_2 = 14.851$$

   Assume that $x_1 = 100$ and $x_2 = -200$ are the quantities of each bond purchased.

   (c) What is the value of the portfolio?

   $$\Pi = -10,927.19$$

   (d) What is the duration of the portfolio?

   $$D_\Pi = 0.1334$$

3. (10) A bond which matures on Dec. 15, 2001 was purchased for $990, with settlement date October 12, 2001. If the bond's coupon rate is 6%, and its face value is $1000, what is its yield to maturity?

accrued interest	=	19.51
invoice price	=	1009.51
yield	=	11.608

Here is the text file of the Maple worksheet I used to generate the solutions to this problem set.

```
Homework Set 6
> restart:
# Problem 1
> M:=49;y:=4;R:=6;x:=183;z:=43;F:=1000:
> ai:=evalf(R*F*(x-z)/(x*200));
> P:=1307.58+ai; #I calculated the flat price on my HP and added it to
> ai.
> for i from 1 to 48 do
> omega[i]:=evalf((R*F/200)*(1+y/200)^(-i+1-z/x)/P): od:
> omega[49]:=evalf((F+R*F/200)*(1+y/200)^(-49+1-z/x)/P):
> duration:=add((k-1+z/x)*omega[k],k=1..49);
> convexity:=add((k+z/x)*(k-1+z/x)*omega[k],k=1..49)/(200+y)^2;
> P_new:=P-(-0.5)*P*duration/(200+y)+(-0.5)^2*P*convexity/2;
M := 49
y := 4
R := 6
x := 183
z := 43
ai := 22.95081967
P := 1330.530820
duration := 28.40280871
convexity := .02717310316
P_new := 1427.674691
> restart:
# Problem 2
# bond 1 calculations
> M1:=3;y[1]:=3;R1:=6;x1:=183;z1:=25;F1:=100;
> ai1:=evalf(R1*F1*(x1-z1)/(x1*200));
> P[1]:=103.13+ai1; #I calculated the flat price on my HP and added it
> to ai1.
> for i from 1 to M1-1 do
> omega_1[i]:=evalf((R1*F1/200)*(1+y[1]/200)^(-i+1-z1/x1)/P[1]): od:
> omega_1[M1]:=evalf((F1+R1*F1/200)*(1+y[1]/200)^(-M1+1-z1/x1)/P[1]):
> duration[1]:=add((k-1+z1/x1)*omega_1[k],k=1..3);
M1 := 3
y[1] := 3
R1 := 6
x1 := 183
z1 := 25
F1 := 100
ai1 := 2.590163934
```

```
P[1] := 105.7201639
duration[1] := 2.052041067
# bond 2 calculations
>  M2:=18;y[2]:=4.0;R2:=5.0;F2:=100.0;
>  P[2]:=add((R2*F2/200)*(1+y[2]/200)^(-k),k=1..M2-1)+(F2+R2*F2/200)*(1+y
>  [2]/200)^(-M2);
>  for i from 1 to M2-1 do
>  omega_2[i]:=evalf((R2*F2/200)*(1+y[2]/200)^(-i)/P[2]): od:
>  omega_2[M2]:=evalf((F2+R2*F2/200)*(1+y[2]/200)^(-M2)/P[2]):
>  duration[2]:=add(k*omega_2[k],k=1..M2);
M2 := 18
y[2] := 4.0
R2 := 5.0
F2 := 100.0
P[2] := 107.4960156
duration[2] := 14.85089709
# portfolio calculations
>  x[1]:=100;x[2]:=-200;
>  PI:=add(x[i]*P[i],i=1..2);
>  duration_port:=(1/PI)*add(x[i]*P[i]*duration[i]/(200+y[i]),i=1..2);
x[1] := 100
x[2] := -200
PI := -10927.18673
duration_port := .1334508197
>  restart:
# Problem 3
>  flat:=990.0;R:=6.0;F:=1000.0;x:=183;z:=64;
>  ai:=R*F*(x-z)/(200*x);#the flat price was obtained from the HP12C
>  P:=flat+ai;
>  eq:=F+R*F/200=P*(1+(y/100)*(z/(2*x)));
>  yield:=solve(eq,y);
flat := 990.0
R := 6.0
F := 1000.0
x := 183
z := 64
ai := 19.50819672
P := 1009.508197
eq := 1030.000000 = 1009.508197 + 1.765260235 y
yield := 11.60837512
>
```