1. (30) Suppose that ABC Corporation's stock is currently selling for \$45 a share. That, $S_u = 56$, $S_d = 42$, and $e^{r\tau} = 1.04$. A broker decides to offer a European put option with expiration at $t = \tau$ and a strike price of \$49.

   (a) What is the numerical value of the delta hedge ratio?

   The option values at expiration time are $U = 0$ and $D = 7$. Thus, the delta hedge ratio equals

$$a = \frac{U - D}{S_u - S_d} = \frac{-7}{56 - 42} = -\frac{1}{2}$$

   (b) Determine the arbitrage free price of this option.

$$\begin{aligned} V_0 &= aS_0 + e^{-r\tau}(U - aS_u) \\ &= -\frac{1}{2}45 + \frac{(1/2)56}{1.04} \\ &\approx 4.423076923 \end{aligned}$$

   (c) A broker sells 100,000 options, and receives 10 cents more than the arbitrage free price for each option sold. The broker decides to hedge his position. How many shares of stock should be bought or sold, and what is the brokers profit when the option expires?

   Since the broker is selling options and a is negative to hedge the position options must be sold, and the number sold is 50,000.
   The amount of cash generated by selling options and stock is

$$\begin{aligned} \text{cash} &= 100,000\,(4.523076923) + 50,000\,(45) \\ &= 2,702,307.69 \end{aligned}$$

   Remember that the selling price is 10 cents more than the arbitrage free price. This money is invested for the time period and grows to

$$2,702,307.69\,(1.04) = 2,810,400.00$$

   At expiration time the brokers position is closed. We'll calculate the value of the portfolio assuming the stock appreciated.
   The put option is worthless and the cost to buy back the shares of stock is

$$50,000\,(56) = 2,800,000$$

   Thus, the profit earned is

$$2,810,400.00 - 2,800,000 = 10,400$$

   Notice that the broker should expect to make this profit as there is an initial profit of 10 cents per option which gives a profit of 10,000, and with interest this equals 10,400.

2. (30) You have been keeping track of closing prices of CSP stock. However, the data you have is not the daily closings but the closings for every other day. Let S_i denote the price of the stock. The average value and standard deviation of the ratio of these prices, S_{i+1}/S_i, are 1.015 and 0.02 respectively.

   (a) Determine the drift parameter μ and volatility σ of this stock.

   The values of μ and σ are calculated as follows. Note that $\triangle t = 2$.

$$\begin{aligned} \mu &= \frac{1.015 - 1}{2} = 0.0075 \\ \sigma &= \frac{.02}{\sqrt{2}} = 0.014 \end{aligned}$$

   (b) You decide to project the next day's stock values using a binomial model with the u, d factors determined by the drift parameter and volatility calculated in part **a**. Assuming that the value of the stock is currently \$65.00, what are S_u and S_d?

   Since we are making a prediction for the next day's stock values we set $\triangle t = 1$ when calculating u and d.

$$\begin{aligned} u &= 1 + \mu\triangle t + \sigma\sqrt{\triangle t} = 1 + 0.0075 + 0.014 = 1.0216 \\ d &= 1 + \mu\triangle t - \sigma\sqrt{\triangle t} = 1 + 0.0075 - 0.014 = 0.9934 \\ S_u &= 65 * 1.0216 = 66.41 \\ S_d &= 65 * 0.9934 = 64.57 \end{aligned}$$

3. (30) The following tree of stock prices was generated by using $u = 1.06$ and $d = 0.9$. Determine the price tree of an American put option with a strike price of \$50. Assume the interest per time period equals $e^{r\tau} = 1.04$.

		56.18
	53	
50		47.70
	45	
		40.5

Remember that the value of an American put in a cell is the maximum of the intrinsic value of the option or the present value of the expected value of the option from the next level of the tree. The probability q is given by

$$q = \frac{e^{r\tau} - d}{u - d} = 0.875$$

The option price tree is shown below

		0
	0.28	
0.83		2.30
	5	
		9.50

4. (10) A one month European put option is currently selling for \$2.50. The strike price of the option is \$50, and the current stock price is \$47. The monthly interest rate is 0.5%. This presents an opportunity for arbitrage. Explain why and how much free profit can be made.

Suppose the option was an American put. This too presents an opportunity for arbitrage. Explain why and how much free profit can be made in this case.

For a European put an arbitrage free profit can be guaranteed by buying the option and also buying a share of stock. The cost for this is \$49.50, and after the option expires the cost is $49.5\,(1.005) = 49.7475$. When the position is closed out possible profits are

$$
\begin{array}{lll}
S \geq 50 & \text{profit equals} & S - 49.75 \geq 50 - 49.75 = 0.25 \\
S < 50 & \text{profit equals} & (50 - S) + S - 49.75 = 50 - 49.75 = 0.25
\end{array}
$$

Thus, no matter what the market does our profit is at least 25 cents.

For an American put we can set up the same portfolio, but then instead of holding it until the option expires we can immediately exercise the option which gives us an immediate profit of $(50 - 47) - 2.50 = 0.50$. Investing this at the current interest rate we are guarantied a riskless profit of

$$0.5\,(1.005) = .5025$$

We could also hold the American option until expiration with the knowledge that we will be assured at least a 25 cent profit and perhaps even more if the stock appreciates to more than \$50.