Sample Questions for Medium Exam 1 Math 314, Spring 2002, February 19

You may use any fact that has been given in class or in the book, as long as you show clearly what you are using. Show your work. Answers are not worth anything without the supporting work.

1. (a) Determine the value of $h$ such that the vectors
\[ \vec{u} = (1, 3, -1), \quad \vec{v} = (-2, -4, 1) \quad \text{and} \quad \vec{w} = (-1, -1, h) \]
are linearly dependent.

(b) For the obtained value of $h$, determine one non-trivial dependence relation involving the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

2. Write down the standard matrix for each of the following linear transformations of the space $\mathbb{R}^3$:
   (a) the transformation $T$ that rotates all the points in $\mathbb{R}^3$ by $\pi/2$ around the $x_3$ axis.
   (b) the transformation $S$ that reflects all the points in $\mathbb{R}^3$ through the plane determined by the $x_2$-axis and the $x_3$-axis (the $x_2x_3$-plane).
   (c) the transformation that first rotates all the points as in part (a) of the problem and then reflects them as in part (b).

3. Let $T$ be a linear transformation such that $T(e_1) = (1, 1, 0)$, $T(e_2) = (0, 1, 1)$ and $T(e_3) = (k, 2, k^2)$. Find all values of $k$ for which the transformation $T$ is not one-to-one.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by
\[ T(x, y) = (2x + y, x + 3y) \]
   (a) Find the standard matrix $A_T$ for the linear transformation $T$. Make it clear how you find this matrix.
   (b) Find the standard matrix for the inverse transformation $T^{-1} : \mathbb{R}^2 \to \mathbb{R}^2$.
   (c) Find the set of points in $\mathbb{R}^2$ that $T$ maps to the $x_1$-axis. In other words, find all points $(a, b)$ such that $T(a, b)$ lies on the $x_1$-axis.
   (d) Write the set you found in part (c) a span of several vectors.

5. Show that, for any three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, the vectors $\vec{u} - \vec{v}$, $\vec{v} - \vec{w}$ and $\vec{w} - \vec{u}$ are linearly dependent.

6. Use the method of row-reduction $[A|I] \sim \cdots \sim [I|A^{-1}]$ to find the inverse of the matrix
\[ A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 4 & -4 \\ 1 & 0 & 4 \end{bmatrix}. \]

7. (a) For which values of the parameter $a$ is the following matrix invertible?
\[ A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix} \]

(b) In the case that $A$ is invertible, find the inverse

8. Let
\[ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}. \]
Find three elementary matrices $E_1, E_2$ and $E_3$ such that $E_3E_2E_1A = I$. 