Small Exam 2  

Math 314, Spring 2002, March 14

You may use any fact that has been given in class or in the book, as long as you show clearly what you are using. Show your work. Answers are not worth anything without the supporting work. I need enough details shown in order to grade your exam.

1. (3pts) The set $S$ consists of the following 5 matrices:

\[
\begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}, \quad \begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix}, \quad \begin{bmatrix}
0 & 0 \\
1 & 1
\end{bmatrix}, \quad \begin{bmatrix}
1 & 0 \\
-1 & 0
\end{bmatrix}, \quad \begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix}.
\]

(a) Determine a basis $B$ of $M_{2 \times 2}$ included in $S$. Note that $M_{2 \times 2}$ is the space of $2 \times 2$ matrices.

(b) Determine the coordinates of

\[
\begin{bmatrix}
1 & 1 \\
2 & 3
\end{bmatrix}
\]

with respect to $B$.

2. (3pts) Let

\[
A = \begin{bmatrix}
1 & 2 & 0 & 3 \\
2 & 4 & -1 & 4 \\
0 & 0 & 1 & 2 \\
0 & 0 & -1 & -2
\end{bmatrix}.
\]

Show that the dimension of $\text{Col}(A)$ is 2. Then find a basis for $\text{Col}(A)$ and a basis for $\text{Nul}(A)$.

3. (2pts) Determine if $U$ is a subspace of $M_{2 \times 2}$, where

(a) $U$ is the set of $2 \times 2$ matrices such that the sum of the entries in the first column is 0.

(b) $U$ is the set of $2 \times 2$ matrices such that the product of the entries in the first column is 0.

Note that you can (and should) use examples to show that something is not a subspace, but you need to make a general argument when you want to show that something is a subspace.

4. (2pts) Warning This problem has nothing to do with calculus or trigonometry. Do not have a faint heart. It is all simple linear algebra at its best.

Consider the space $V$ of functions spanned by the functions in $B = \{\cos^2(x), \sin^2(x)\}$. In what follows, you may assume that the functions in $B$ are independent, i.e., form a basis for $V$. As an example, one function in the space $V = \text{span}(B)$ is $\cos(2x)$, which can be easily seen from the following equality

\[
\cos(2x) = \cos^2(x) - \sin^2(x)
\]

(a) Use the well known formula

\[
1 = \cos^2(x) + \sin^2(x)
\]

to show that $f(x) = 1$ is also a function in $V$ and find its coordinates with respect to $B$.

(b) Use the coordinates of the functions $\cos(2x)$ and 1 with respect to $B$ to show that these functions form another basis for $V$. 