

Texas A&M University
Department of Mathematics
Groups and Dynamics Seminar

Some finiteness results in algebraic geometry for a
free group

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We introduce some basic objects of algebraic geometry for groups similar to the ones from commutative algebra: a system of equations, an algebraic variety, a radical, a coordinate group, a Zariski topology and so on. Suppose we have two irreducible systems of equations $T(X) = 1$ and $S(X, Y) = 1$ over a free group. Let H be the coordinate group of the first system, K be the coordinate group of the second system. Then it is known that K and H are fully residually free groups. Suppose that H is embedded into K . We say that K does not have a sufficient splitting modulo H if K either does not split as an amalgamated free product (HNN extension) with abelian edge group and H being elliptic or it splits but does not split as a free product and “minimal” solutions of $S(X, Y) = 1$ with respect to the group of automorphisms of K corresponding to these splittings define the algebraic variety with the same coordinate group K . (Example: $X = \{x\}, Y = \{y_1, z_1, \dots, y_m, z_m\}$, $T(x)$ is trivial, $S(X, Y) = x \prod_{i=1}^m [y_i, z_i]$.) We introduce the notion of solutions of finite type of the system $S(X, Y) = 1$, and will show that one can effectively find a constant N such that for every solution of $T(X) = 1$ there are at most N finite type solutions of $S(X, Y) = 1$.

As one of the applications we will show that the number of finite type solutions of quadratic equations depends only on the number of variables and does not depend on the coefficients. This is interesting to compare with Grigorchuk-Kurchanov, Comerford-Edmunds description of algebraic varieties for quadratic equations in free groups.

These are joint results with A. Myasnikov.