Texas A&M University
Department of Mathematics
Groups and Dynamics Seminar

The idea of slenderness

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27th of October, 2004, 3:00-3:50, 216 Milner Hall

The well known expression of the universal properties of the product construction may be given via the following natural isomorphism, true for every object M, index set I and family of objects A_i, $i \in I$:

$$Hom_{\mathcal{C}}\left(M, \prod_{i\in I} A_i\right) \cong \prod_{i\in I} Hom_{\mathcal{C}}(M, A_i), \qquad f \mapsto (\pi_i f)_{i\in I}, \qquad (f_i)_{i\in I} \mapsto \prod_{i\in I} f_i.$$

By reversing some of the arrows we ultimately arrive at a (questionabe) isomorphism

$$\bigoplus_{i\in I} Hom_{\mathcal{C}}(A_i, M) \cong Hom_{\mathcal{C}}\left(\prod_{i\in I} A_i, M\right), \qquad (f_i)_{i\in I} \mapsto \sum_{i\in I} f_i \pi_i$$

that is not going to be fulfilled for all objects M (or perhaps all index sets I). For instance, one can see that if M is a finite group, or the set of rationals, the latter isomorphism does not hold. The problem is to find all objects M such that, for every (say) countable family A_i, $i \in \mathbb{N}$, the latter isomorphims holds. This innocent question has deep ramifications connecting several areas of mathematics and I want to give the audience a glimpse of this beauty.