Growth in Iterated Monodromy Groups

(Joint work with Rodrigo Perez)

Abstract

Iterated Monodromy Groups stem from complex dynamics. They are groups of automorphisms of infinite regular rooted trees and are similar to Grigorchuk's group. Let T be the infinite binary rooted tree, in which every non-root vertex has precisely three neighbors: its parent, its left child, and its right child. The automorphism group W of T admits a wreath product decomposition

$$W = W \wr \mathbf{Z}_2 = (W \times W) \rtimes \mathbf{Z}_2$$

where $\mathbf{Z}_2$ is generated by the "swap" automorphism σ that interchanges the left and right subtree. This decomposition can be used to describe automorphism by equations or systems of equations. E.g., the equation

$$\xi = (\xi, 1)\sigma$$

has a unique solution: The s on to the right tells us that ξ swaps the children of the root. With this knowledge, we can see how $\xi = (\xi, 1)\sigma$ acts on the grandchildren of the root. Iterating the procedure specifies the automorphism x.

I will discuss the intermediate growth of the following three groups:

a) Grigorchuk's group $G = \langle \sigma, \alpha, \beta, \gamma \rangle$ where:

$$\begin{aligned} \alpha &= (\beta, \sigma) \\ \beta &= (\gamma, \sigma) \\ \gamma &= (\alpha, 1) \end{aligned}$$

b) The group $H = \langle \sigma, \alpha, \beta \rangle$ where:

$$\begin{aligned} \alpha &= (\beta, \sigma) \\ \beta &= (\alpha, 1) \end{aligned}$$

c) The group $I = \langle \sigma, \alpha, \beta \rangle$ where:

$$\begin{aligned} \alpha &= (\beta, \sigma) \\ \beta &= (1, \alpha) \end{aligned}$$

This group is the iterated monodromy group for $z^2 + i$. Intermediate growth of this group has been claimed by V. Nekrashevych.

I will present a more or less unified approach to the intermediate growth for all these groups: each group is just a little more involved than the previous one and requires a little refinement in the argument.