

PERCOLATION AND AMENABILITY OF CAYLEY GRAPHS

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The talk will be devoted to the discussion of *percolation* on different infinite graphs. Bernoulli percolation in \mathbb{Z} is an important phenomenon and has been extensively studied. It was noticed by several authors (see the work of Lyons, Benjamini, Schramm) that the study of percolation is also of interest on other graphs. One of the starting points of this research was the work of Grimmet and Newman who observed that the percolation in $\mathbb{Z} \times$ *an infinite tree* differs dramatically from that in the lattice. Namely, there exist values of the parameter p for which infinitely many infinite clusters coexist. An argument of Burton and Keane shows that such phenomenon cannot occur in an amenable Cayley graph. Benjamini and Schramm conjectured that the converse is true, namely that the nonuniqueness phase of percolation is nonempty in any nonamenable Cayley graph.

There exist many equivalent definitions of amenability. For the purposes of this talk the following geometric definition will be useful. A finitely generated group G is amenable if for any finite generating set S , $\inf |\partial A|/|A| = 0$, where the infimum is taken over all finite subsets of G , and the boundary ∂A of A is the set of edges in the Cayley graph of (G, S) with one end in A and other in the complement of A .

I will discuss some general properties of the Bernoulli bond percolation in both amenable and nonamenable Cayley graphs. I will then report on a joint work with Igor Pak in which we prove a weaker version of the conjecture of Benjamini and Schramm, namely that every finitely generated nonamenable group has a finite generating set, such that the percolation on the corresponding Cayley graph exhibits a nonempty nonuniqueness phase.

Together with the result on amenable groups mentioned above, this gives the following new characterization of amenability of finitely generated groups. A finitely generated group G is amenable if and only if the nonuniqueness phase of percolation on $\Gamma(G, S)$ is empty for every finite generating set S .