Diophantine Properties of IETs

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(joint work with Jon Chaika)

Let $X = [0,1)$ and λ stand for the unit interval and the Lebesgue measure on it. We present shrinking targets results for IETs (interval exchange transformations). One of the results is that if an IET (X,T) is λ-ergodic, then the equality

$$\liminf_{n\to\infty} n\,|T^n(x) - y| = 0, \tag{1}$$

holds for $\lambda \times \lambda$ almost all pairs $(x,y) \in X^2$.

A secial case of this result, for minimal 2-IETs (irrational rotations), is already known (J.Tseng, D.Kim). The factor n in (1) cannot be replaced by one approaching infinity faster, even in the case of 2-IETs.

The result may fail for some minimal but not λ-ergodic 4-IETs.

The above result is contrasted with the following one. For a "random" 3-IET T, for all $\alpha > 0$ and Lebesgue a.a. $x, y \in X$ the equality

$$\liminf_{n\to\infty} n^\alpha\,|T^n(x) - T^n(y)| = \infty \tag{2}$$

holds (even though $\liminf_{n\to\infty} |T^n(x) - T^n(y)| = 0$ (for a.a. $x, y \in X$) because a "random" IET is weakly mixing and hence $T \times T$ is ergodic.

Related open questions will be presented.