

MATH 151 FINAL EXAM, SPRING, 2006

FORM A

PRINT LAST NAME:

FIRST NAME:

SIGNATURE:

ID NUMBER:

INSTRUCTOR'S NAME:

SECTION #:

INSTRUCTIONS:

Mark the correct choice on your ScanTron form using a #2 pencil. Please, also record your choice on the exam! After each question exactly one answer is supposed to be correct. (However, if you find that none of the answers is correct or there are at least two correct options listed to answer a question, write clearly on your exam sheet something like “none of the answers is correct” or “there are at least two correct options listed”.) The exams and ScanTrons will be collected after two hours. Each problem is worth 4 points; NO partial credit will be given. Calculators and books are NOT allowed during the exam. The problems are designed so that all calculations can be carried out by hand.

1. What is the limit of

$$\frac{\sqrt{2x+22}-4}{x+3}$$

when x approaches -3 ?

- (a) The limit does not exist since $f(-3)$ is undefined.
- (b) The limit is ∞ .
- (c) The limit is $1/2$.
- (d) The limit is $1/4$.
- (e) The limit is $1/8$.

2. Evaluate

$$\lim_{x \rightarrow 0} (1 - 4 \sin x - 3x^2)^{2/x}.$$

Hint: First take the natural logarithm of the function.

- (a) e^{-8}
- (b) $1/e^6$
- (c) e^{-3}
- (d) e^2
- (e) e^8

3. What is the domain of the function $f(x) = \ln(2 + \ln x)$?

- (a) $x > 2e$
- (b) $x < e^{-2}$
- (c) $x > e^{-2}$
- (d) $x > 2$
- (e) $x > e^2$

4. Evaluate

$$\lim_{x \rightarrow \infty} \arctan(\ln x - x).$$

- (a) 0 (b) $-\pi/2$ (c) $\pi/2$ (d) ∞ (e) $-\infty$

5. Let $f(x) = \arctan(2 \ln x)$. Then $f'(e) =$

- (a) $\frac{2}{5}$ (b) $\frac{2e}{5}$ (c) $\frac{2}{5e}$ (d) $\frac{-2}{5}$ (e) $\frac{1}{5e}$

6. Express

$$\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left(2 + \frac{5i}{n}\right)^6$$

as a definite integral.

(a) $\frac{3}{5} \int_2^7 (2+t)^6 dt$ (b) $\int_2^7 t^6 dt$ (c) $5 \int_2^7 t^6 dt$

(d) $\frac{3}{5} \int_0^5 (2+t)^6 dt$ (e) $3 \int_2^7 t^6 dt$

7. Let

$$f(x) = \int_2^x \frac{t^2 - \ln t}{3 + \ln t} dt.$$

Then $f'(e) =$

- (a) $\frac{e^2 - 1}{3}$ (b) $\frac{e^2 - 1}{4}$ (c) $\frac{e^2}{4}$ (d) $\frac{e^2}{3}$ (e) $\frac{e^2}{3 + e}$

8. An antiderivative of

$$f(x) = \frac{1}{x(\ln x)(\ln(\ln x))}$$

on (e, ∞) is:

- (a) $\ln(\ln(\ln x))$ (b) $-\ln(\ln(\ln x))$ (c) $\ln(\ln x)$ (d) $-\ln(\ln x)$ (e) $(\ln(\ln x))^2$

9. Let $f(x) = |2x - 9|$. Which one is true about the absolute minimum value of $f(x)$ on the interval $[-4, 4]$?

- (a) it is attained at -4 .
(b) it is attained at 4 .
(c) it is attained at a point c where $f'(c)$ does not exist.
(d) it is attained at a point c where $f'(c) = 0$.
(e) it is not attained on $[-4, 4]$.

10. There are two tangent lines to the parabola $y = 2x^2$ going through the point $(0, -98)$. The x coordinate of the touching point of one of these lines is

- (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

11. Suppose that f is a differentiable function on an open interval containing 2 such that $x(f(x))^3 + f(x) = 57$ and $f(2) = 3$. Find the value of $f'(2)$.

- (a) $\frac{-1}{2}$ (b) $\frac{-27}{61}$ (c) $\frac{-15}{55}$ (d) $\frac{10}{39}$ (e) $\frac{-27}{55}$

12. Let

$$\mathbf{r}(t) := \frac{t^2 + 2t - 15}{t - 3} \mathbf{i} + \frac{t^2 + 2t - 15}{t + 5} \mathbf{j}$$

be a vector-valued function. What is

$$\lim_{t \rightarrow 3} \mathbf{r}(t) ?$$

- (a) $8\mathbf{i}$ (b) $8\mathbf{j}$ (c) 8 (d) $-8\mathbf{j}$ (e) $(t + 5)\mathbf{i} + (t - 3)\mathbf{j}$

13. Which line below is a horizontal asymptote for

$$f(x) := \frac{\sqrt{256x^2 + 5x + 3}}{4x - 3}?$$

- (a) $y = 2$ (b) $y = 4$ (c) $y = 8$ (d) $y = -6$ (e) $y = -16$

14. Let

$$f(x) = \begin{cases} 5x^2 + 3x, & x < 2 \\ x^2 + 5x + 4c, & x \geq 2. \end{cases}$$

Suppose f is continuous on the real line. Which one of the following is true?

- (a) $c = 3$ (b) $c = 4$ (c) $c = 2$ (d) c can be arbitrary
(e) there is no such c

15. Let

$$f(x) = \begin{cases} 5(x^3 - x), & x < 3 \\ c(x - 3) + 120, & x \geq 3. \end{cases}$$

Suppose f is differentiable on the real line. Which one of the following is true?

- (a) $c = 140$ (b) $c = 130$ (c) $c = 120$ (d) c can be arbitrary
(e) there is no such c

16. Let $f(x) = 5e^{3x} + 3 \arcsin x$. The inverse function of f is denoted by $g = f^{-1}$. Then the value of $g'(5)$ is

- (a) $1/18$ (b) $1/15$ (c) undefined (d) $1/3$ (e) $1/5$

17. Consider the curve C with parametric equations

$$\begin{cases} x = t^2 - 2t \\ y = t^3 + t - 9. \end{cases}$$

Then the slope of the tangent line to C at the point $P(3, 21)$ is

- (a) 7 (b) 14 (c) $1/7$ (d) $1/14$ (e) 28

18. Find the linear approximation of $f(x) = x^{2/3} - 3$ near 27.

- (a) $6 - (2/3)(x - 27)$ (b) $6 - (2/9)(x - 9)$ (c) $6 + (2/9)(x - 9)$
(d) $6 + (2/9)(x - 27)$ (e) $9 + (2/9)(x - 27)$

19. Suppose $f'(5) = 6$. What is

$$\lim_{h \rightarrow 0} \frac{f(5 - 3h) - f(5)}{h} ?$$

- (a) -18 (b) 6 (c) -6 (d) -2 (e) 2

20. What is the exact value of

$$\lim_{x \rightarrow \infty} \left(\sqrt{9x^2 + 8x} - 3x \right) ?$$

- (a) $1/3$ (b) $-1/3$ (c) $2/3$ (d) $4/3$ (e) the limit does not exist

21. What is the exact value of

$$\int_{66}^{99} \frac{1}{x} dx ?$$

- (a) $\ln 1.5$ (b) $\ln 33$ (c) $99^{-2} - 66^{-2}$ (d) $66^{-2} - 99^{-2}$ (e) $\ln 99 / \ln 66$

22. Suppose g is the antiderivative of a continuous function f on the real number line. Suppose $g(2) = 3$ and $g(5) = -9$. What is the exact value of

$$\int_2^5 (3f(x) + 2x) dx ?$$

- (a) -36 (b) -12 (c) -33 (d) -17 (e) -15

23. Suppose $f''(x) = (x - 2)^4(\cos x)(3 - \sin x)$. Then f has points of inflection in $[-4, 4]$ at $x =$

- (a) 2 and $\pi/2$ only (b) $-\pi/2$ and $\pi/2$ only (c) $-\pi/2, \pi/2,$ and 2 only
(d) 2 and 3 only (e) $\pi/2$ only

24. Let

$$f(x) = \frac{2x + 3}{6x^2 + 5x - 6}.$$

Which of the following statements is true?

- (a) f has no vertical asymptotes.
(b) f has vertical asymptotes at $x = -3/2$ and $x = 2/3$.
(c) f has vertical asymptote at $x = -3/2$ only.
(d) f has vertical asymptote at $x = 2/3$ only.
(e) $y = 1$ is a horizontal asymptote of f .

25. Let $f(x) = |\sin(5x)|$. Then the number of solutions to $f(x) = 1/3$ on $[-\pi, \pi]$ is exactly

- (a) 4 (b) 8 (c) 12 (d) 10 (e) 20