

## Math 142, Spring 2004

## Section 5.2 Exponential Functions and Their Derivatives

**1. Composite Functions:** A function  $h(x)$  is called a composition of functions  $f$  and  $g$  if

$$h(x) = f[g(x)]$$

In other words  $x$  is the argument (an independent variable) of  $g(x)$ , and  $g(x)$  is the argument (an independent variable) of  $f(x)$ . Thus the domain of  $h(x)$  is the set of all real value of  $x$  such that  $x$  is in the domain of  $g$  and  $g(x)$  is in the domain of  $f$ .

**Example 1 (Also see Ex.1, p. 314).** Let  $f(u) = 2u$  and  $g(x) = e^x$ . Find  $f[g(x)]$  and  $g[f(u)]$ .

**Example 2 (Also See Ex.2, p. 314).** Write each function as a composition of two simpler functions.

(a)  $y = 50e^{-2x}$

(b)  $y = \sqrt[3]{1 + x^3}$

(!) **Note** that there are multiple ways of representing a function as a composition of simpler functions.

**2. The Derivative of  $e^x$  (without proof):**  $(e^x)' = e^x$  or  $\frac{d}{dx}e^x = e^x$ .

**Example 3 (Also see Ex.3, p. 316)** Find  $f'(x)$  for

(a)  $f(x) = (x^2 - 2e^x)^3$

(b)  $f(x) = x^2e^x$

(!) **Be careful** not to apply the power rule while differentiating  $e^x$ , i.e.  $(e^x)' \neq xe^{x-1}$ .

**3. General Exponential Derivative Rule:** If  $u(x)$  is a function of  $x$ , then

$$(e^{u(x)})' = e^{u(x)}u'(x)$$

**Example 4 (Also see Ex. 4, p.317).** Find  $f'(x)$  for

(a)  $f(x) = e^{-x^3}$

(b)  $f(x) = \frac{e^{2x}}{1 + e^{2x}}$

**4. Graphing exponential functions:** The graphs of the two basic exponential functions  $f(x) = e^x$  and  $g(x) = e^{-x} = \frac{1}{e^x}$  are given in the figure below.

**Fig. 1**

**Example 5 (Also see Ex. 5, p. 319).** *Analyze the function  $f(x) = xe^{-0.5x}$  and sketch the graph.*

If time allows ...

**Example 6 (Also see Ex. 6, p. 320).** *An Internet store sells reversible fleece blankets. If the store sells  $x$  blankets at a price of  $\$p$  per blanket, then the price-demand equation is  $p = 200e^{-0.002x}$ .*

- *Find the rate of change of price with respect to demand when the demand is 400 blankets and interpret.*
- *How much should the store charge for reversible fleece blankets in order to maximize revenue?*