# Texas Geometry and Topology Conference
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**Yakov Eliashberg, Stanford University,** ***Orderability of groups of contact transformations and related topics***

There is a natural candidate for a partial order on the universal cover $G(M, \xi)$ of the group of contact diffeomorphisms of a contact manifold $(M, \xi)$. Namely, we say that $f > g$ if $fg^{-1}$ can be generated by a positive Hamiltonian function. However, it is not clear whether this order is non-trivial. Examples of *orderable* contact manifolds, e.g. manifolds for which this order is non-trivial were given by A. Givental in [3] (odd-dimensional real projective spaces) and by L. Polterovich and the author in [2] (the unit cotangent bundle of a torus). In a more recent paper [1] of S.-S. Kim, L. Polterovich and the author there were discovered a class of non-orderable contact manifolds which included standard contact spheres. It was also shown that the orderability problem is tightly related with many other interesting contact-geometric problems, and in particular, with a contact analog of Gromov's famous non-squeezing theorem (1985) which states that the standard symplectic ball cannot be symplectically squeezed into the cylinder of smaller radius. It was shown in this paper that the contact non-squeezing phenomenon exists on large scales, but it disappears on small scales. In the talk there will be discussed these results as well as more recent progress in this direction, in particular by I. Milin and G. Ben-Simon. We will also discuss connections with other problems, such as existence of quasi-morphisms on $G(M, \xi)$.

## References


[1] Y. Eliashberg and L. Polterovich, Partially ordered groups and geometry of contact transformations, *GAFA*, 1 (2000), 1448–1476.

[2] Y. Eliashberg, S.-S. Kim, L. Polterovich, Geometry of contact transformations: orderability versus squeezing, *Geom. and Topol.*, 10 (2006), 1635–1748.

[3] A. Givental, Nonlinear generalization of the Maslov index, in *Theory of singularities and its applications*, pp. 71-103, Adv. Soviet Math., 1, Amer. Math. Soc., Providence, RI, 1990.