# Texas Geometry and Topology Conference
# Meeting 38. Texas A&M University, October 19-21, 2007

**Natasa Sesum, Columbia University,** ***Compactness results for the Kähler-Ricci flow***

We consider the Kähler-Ricci flow $\frac{\partial}{\partial t}g_{i\bar{j}} = g_{i\bar{j}} - R_{i\bar{j}}$ on a compact Kähler manifold $M$ with $c_1(M) > 0$, of complex dimension $k$. We prove the $\epsilon$-regularity lemma for the Kähler-Ricci flow, based on Moser's iteration. Assume that $|\mathrm{Ric}|$ and $\int_M |\mathrm{Rm}|^k \, dV_t$ are uniformly bounded along the flow. Using the $\epsilon$-regularity lemma we derive the compactness result for the Kähler-Ricci flow. Under our assumptions, if $k \geq 3$ in addition, using the compactness result we show that $|\mathrm{Rm}| \leq C$ holds uniformly along the flow. This means the flow does not develop any singularities at infinity. We use some ideas of Tian from to prove the smoothing property in that case.

To be more precise we first prove the following result.

**Theorem 1.** *Let $g(t)$ be the Kähler-Ricci flow on a compact, Kähler manifold $M$, with $c_1(M) > 0$, with $\int_M |\mathrm{Rm}(g(t))|^{n/2} dV_{g(t)} \leq C$ and $|\mathrm{Ric}| \leq C$ along the flow. Then for every sequence $t_i \to \infty$ there is a subsequence so that $(M, g(t_i + t))$ converges to $(M_\infty, g_\infty(t))$, where*

*(a) $M_\infty$ is an orbifold with finitely many isolated singular points, $\{p_1, \ldots, p_N\}$, and the convergence is smooth outside those singular points.*

*(b) The limit metric $g_\infty$ is a Kähler-Ricci soliton in an orbifold sense, that is, satisfies the Kähler-Ricci soliton equation,*

$$\begin{aligned} (g_\infty)_{i\bar{j}} - R_{i\bar{j}}(g_\infty) &= \partial_i \partial_{\bar{j}} f_\infty, \\ \partial_{\bar{i}} \partial_{\bar{j}} f_\infty &= 0, \end{aligned} \tag{1}$$

*off the singular points. Moreover, for every singular point $p_j$, there is a neighbourhood in $M_\infty$ which lifts to an open set $D_j \subset \mathrm{C}^{n/2}$, and the lifting of an orbifold metric $g_\infty$ satisfies equivalent equations to (1) in $D_j$.*

By using the previous compactness theorem for the Kähler-Ricci flow we prove the following.

**Theorem 2.** *Let $g(t)$ be a Kähler-Ricci flow on a compact, Kähler manifold $M$ of complex dimension $k$ ($n = 2k$), with $c_1(M) > 0$ and with $\int_M |\mathrm{Rm}(g(t))|^{n/2} dV_{g(t)} \leq C$ along the flow. Then if $k \geq 3$, the curvature operator is uniformly bounded along the flow.*

Theorem 2 tells us the blow up of curvature does not happen at infinity which means we could potentially talk about smooth limits of the flow.