Texas Geometry and Topology Conference

This is a report on the presentations at the 40th meeting of the Texas Geometry and Topology Conference at University of Texas at Austin, October 10-12, 2008. This conference was partially supported by National Science Foundation Grant DMS-0605082 and the University of Texas at Austin. Speakers reported on recent research. For this report, speakers have provided synopses of their talks together with broader discussions of the significance and context of their results.

Meeting 40. University of Texas at Austin, October 10-12, 2008

Lewis Bowen, University of Hawaii, *Free subgroups of lattices*

The surface subgroup conjecture posits that if Γ is a cocompact Kleinian group then Γ contains a surface subgroup; that is, a subgroup isomorphic to the fundamental group of a closed surface of genus at least 2. This is implied by the Virtual Haken conjecture (due to Thurston) and is of central importance to the classification of hyperbolic 3-manifolds.

Here is a more general problem. Let Γ be a discrete cocompact subgroup of G, a locally compact unimodular group. Prove the existence of subgroups of Γ satisfying prescribed properties. These properties may be of a group theoretic nature or an asymptotic-geometric nature.

The point behind this problem is that it might be possible to use knowledge of G to infer the existence of certain kinds of subgroups of Γ. This talk (and the paper [Bo07]) explores the following strategy for attacking this problem: given a subgroup $F < G$, we attempt to "perturb" F slightly so that some finite-index subgroup of the perturbed group lies inside Γ. For this to be successful, we would like the perturbed subgroup to have isomorphism type close to that of F and to have asymptotic geometric properties close to that of F.

To be precise, let F be an abstract group and $\phi : F \to G$ a homomorphism. Let $S \subset F$ be a finite symmetric generating set. Let d be a left-invariant proper metric on G. For $\epsilon > 0$, we will say that a map $\phi_\epsilon : F \to G$ is an ϵ-**perturbation** of F if

$$d(\phi_\epsilon(g)^{-1}\phi_\epsilon(gs), \phi(s)) = d(\phi_\epsilon(gs), \phi_\epsilon(g)s) \leq \epsilon$$

for all $g \in F$ and $s \in S$. ϕ_ϵ need not be a homomorphism. Indeed, we do not even require that it maps the identity element to the identity element.

For example, if $G = \mathbb{R}$, $F = \mathbb{Z}$ and $\phi : \mathbb{Z} \to \mathbb{R}$ is the homomorphism $\phi(n) = n\tau$ for some number $\tau > 0$ then $\phi_\epsilon : \mathbb{Z} \to \mathbb{R}$ need only satisfy $|\phi_\epsilon(n+1) - \phi_\epsilon(n) - \tau| \leq \epsilon$ for all $n \in \mathbb{Z}$.

We say that $\phi_\epsilon : F \to G$ is *virtually a homomorphism into* Γ if there exists a finite index subgroup $F' < F$ such that

$$\phi_\epsilon(f_1 f_2) = \phi_\epsilon(f_1)\phi_\epsilon(f_2) \qquad \forall f_1 \in F',\ f_2 \in F$$

and $\phi_\epsilon(F') < \Gamma$.

Theorem 1. *(Main theorem) Suppose G, d, F, S and ϕ are as above. Suppose F is free, finitely generated and S is a symmetric free generating set for F. Then for every $\epsilon > 0$ there exists an ϵ-perturbation ϕ_ϵ of ϕ that is virtually a homomorphism into Γ.*

I do not know if the theorem remains true if F is required to be a surface group instead.

Asymptotic Geometry

Claim: if ϵ is small and F is "nice" then an ϵ-perturbation of F does not change F very much. To be specific, assume that $G = SO(n,1)$ and F is a convex cocompact free subgroup of G.

Let $\mathbb{H}^n$ denote hyperbolic n-space. So G is the group of orientation preserving isometries of $\mathbb{H}^n$. Let S^{n-1}_∞ denote the boundary at infinity. For any $p \in \mathbb{H}^n$, let $Fp = \{fp : f \in F\}$. The closure of Fp in $\mathbb{H}^n \cup S^{n-1}_\infty$ is denoted by $\overline{Fp}$. Its intersection with S^{n-1}_∞ is the *limit set of* F, denoted $L(F)$. It does not depend on p. Let $D(F)$ denote its Hausdorff dimension.

Similarly, if ϕ_ϵ is an ϵ-perturbation of the inclusion map $F \to G$, let $\phi_\epsilon(F)p = \{\phi_\epsilon(f)p : f \in F\}$ and let $L(\phi_\epsilon(F))$ be the intersection of S^{n-1}_∞ with the closure $\overline{\phi_\epsilon(F)p}$. Let $D(\phi_\epsilon(F))$ denote its Hausdorff dimension.

Theorem 2. *Let $F < SO(n,1)$ be a free convex cocompact subgroup. For every $\epsilon > 0$ let ϕ_ϵ be an ϵ-pertubation of the inclusion map $F \to G$. Then,*

1. *for all $\epsilon > 0$ sufficiently small, ϕ_ϵ is 1-1;*
2. *if $\phi_\epsilon(id) = id$ (where id denotes the identity element) then $L(\phi_\epsilon(F))$ converges to $L(F)$ in the Hausdorff topology as $\epsilon \to 0$.*

Moreover, if ϕ_ϵ is a virtual homomorphism then $D(\phi_\epsilon(F)) \to D(F)$ as $\epsilon \to 0$.

Applications

If H is any subgroup of $G = SO(n,1)$, let $D_{free}(H)$ denote the set of all numbers of the form $D(F)$ where F is a free, convex cocompact subgroup of H.

Theorem 3. *If Γ is a lattice in $G = SO(n,1)$ then $\overline{D_{free}(\Gamma)} = \overline{D_{free}(G)}$.*

Remark. It is easy and well-known that $D_{free}(SO(2,1)) = (0,1)$. From work of Thurston and others on geometrically infinite free groups it can be proven that $D_{free}(SO(3,1)) = (0,2)$. It is not known whether these results extend to $SO(n,1)$ for $n \geq 4$.

The Cheeger constant of a closed Riemannian manifold M is defined by

$$h(M) := \inf_S \frac{area(S)}{\min(vol(X_1), vol(X_2))},$$

where S varies over all codimension 1 submanifolds that divide M into two pieces, X_1 and X_2. If M is noncompact then

$$h(M) := \inf_X \frac{area(\partial X)}{vol(X)}.$$

where X varies over all compact codimension 0 submanifolds of M.

Theorem 3 was recently employed by Lackenby, Long and Reid [LLR08] to obtain the next two theorems.

Theorem 4. *If M is a closed hyperbolic 3-manifold then there exists a sequence of infinite-sheeted coverings M_i of M such that $h(M_i) \to 0$.*

Theorem 5. *If M is a closed hyperbolic 3-manifold and $\pi_1(M)$ is LERF then there exists a sequence of finite coverings M_i of M such that $h(M_i) \to 0$. I.e., $\pi_1(M)$ does not have property τ. I.e., the Lubotzky-Sarnak conjecture holds for $\pi_1(M)$.*

Theorem 3 was recently used by Lackenby [La08] to prove:

Theorem 6 (Lackenby)**.** *If $\Gamma < SO(3,1)$ is discrete, finitely generated and contains a noncyclic finite subgroup then either Γ is finite, Γ is virtually free or Γ contains a surface subgroup.*

References

[Bo07] L. Bowen. *Free Groups in Lattices*. arXiv:0802.0185, submitted to Geometry and Topology.

[La08] M. Lackenby. *Surface subgroups of Kleinian groups with torsion*. arXiv:0804.1309.

[LLR08] M. Lackenby, D. Long and A. Reid. *LERF and the Lubotzky-Sarnak conjecture*. Geom. Topol. 12 (2008), 2047–2056.

Jeff Cheeger, Courant Institute, *Differentiable Structures on Metric Measure Spaces*

A classical theorem of Rademacher asserts that a real valued Lipschitz function on $\mathbf{R}^n$ is differentiable almost everywhere with respect to Lebesgue measure. The statement incorporates three notions:

- *Lipschitz,* which is defined for any metric space, (X, d^X).
- *Almost everywhere,* which is defined for any measure space, (X, μ).
- *Differentiability,* which classically was defined only for real-valued functions on spaces which are locally bi-Lipschitz to $\mathbf{R}^n$.

In the late 1990's we gave a notion of (first order) *differentiable structure* on any metric measure space (X, d^X, μ). When it exists, this structure is unique up to bi-Lipschitz equivalence. We showed that if the measure μ is doubling and a Poincaré inequality holds in the upper gradient sense, then such a differentiable structure exists. The Poincaré inequality is equivalent to the existence of "sufficiently many" curves of finite length (in a sense which can be made precise). We we call such spaces *PI spaces.*

Examples of spaces of PI spaces include fractals such as Carnot groups with Carnot–Caratheodory metrics and Laakso spaces. Although these spaces are not infinitesimally bi-Lipschitz to $\mathbf{R}^n$, we showed they do possess a finite dimensional (measurable) cotangent bundle which is the receiving space for differentials of Lipschitz functions. Classical fractals such as the Sierpinski gasket are not PI spaces because they do not contain sufficiently many curves of finite length (although every pair of points is joined by a minimal geodesic).

The theory has had applications which are not internal to the theory itself. Here we emphasize the connection with bi-Lipschitz nonembedding theorems. We showed (roughly speaking) that a PI space which is bi-Lipschitz to a subset of some $\mathbf{R}^N$ must look quite "tame". In particular, the above mentioned fractal spaces do not bi-Lipschitz embed in $\mathbf{R}^N$ for any $N < \infty$.

The above suggests the problem of giving conditions under which a given PI space does or does not bi-Lipschitz embed in particular infinite dimensional Banach spaces. This question, which is of considerable interest in theoretical computer science, has been addressed in joint work with Bruce Kleiner.

It was clear initially, that if one could extend the differentiation theory to Lipschitz functions, $f : X \to V$, for some Banach space V, then the bi-Lipschitz nonembedding theorem (and its proof) would carry over as well. This was done several years ago for separable dual space targets. Recently, we proved the optimal possible result. Namely, let V have the property that all Lipschitz functions $f : \mathbf{R} \to V$ are differentiable. (In classical terminology, V is said to possess the *Radon–Nikodym Property*.) Then all Lipschitz functions $f : X \to V$ are differentiable for any PI space X. The proof of this theorem led to the uncovering of additional ways in which PI spaces resemble $\mathbf{R}^n$ at the infinitesimal level (although, once again, such spaces can have fractional Hausdorff dimension).

There is a simple classical example of a Lipschitz map $\mathbf{R} \to L^1$ which is *not* differentiable anywhere, i.e. L^1 does not have the Radon-Nikodym Property. Moreover, in the context of theoretical computer science, questions of bi-Lipschitz embedding in L^1 play a distinguished and important role. For the domains

$\mathbf{R}^n$ and the Heisenberg group equipped with its Carnot–Caratheodory metric, Kleiner and I showed (their possible nondifferentiability notwithstanding) that Lipschitz functions to L^1 *are* differentiable in a weakened sense. For the Heisenberg group this *does* preclude the existence of a bi-Lipschitz embedding in L^1. Our result is based on a connection between Lipschitz maps $f : X \to L^1$ and subsets of X with *finite perimeter*. (These are classical objects of study in geometric measure theory). The notion of differentiability and the connection with sets of finite perimeter are new even for the domain $\mathbf{R}^n$.

The above work gave a natural counter example to the Goemans–Linial conjecture of theoretical computer science. (The first such was given slightly earlier by Khot–Vishnoi.). In joint work with Kleiner and Assaf Naor, we show that this counter example is qualitatively stronger than the previous one and in fact, is qualitatively close to being best possible.

Tom Farrell, Binghamton University , *The Space of Negatively-Curved Metrics on a Closed Manifold*

The talk was a report on joint work with Pedro Ontaneda.

Let M be a closed (connected) smooth manifold of dimension n. Let $\mathrm{Met}(M,-)$ denote the space of all Riemannian metrics on M whose sectional curvatures are all negative. And let $C(p)$ denote the infinite abelian group, which is the countable direct sum of cyclic groups of prime order p.

Theorem. *Assume $n > 9$ and $\mathrm{Met}(M,-)$ is nonempty. Then:*

1. *$\mathrm{Met}(M,-)$ has infinitely many path components K.*
2. *For each component K and each prime $p > 2$, $C(p)$ is a subgroup of the $2p-4$ homotopy group of K provided that $n > 4p+2$.*
3. *Also, $C(2)$ is a subgroup of the fundamental group of K provided $n > 13$.*

In particular, Part 1 of the Theorem solves “Question 7.1” on the 1984 list compiled by Burns and Katok about non-positively curved manifolds.

The Teichmüller and Moduli spaces of M are quotient spaces of $\mathrm{Met}(M,-)$ obtained by identifying isometric metrics in the case of the Moduli space and identifying metrics isometric via isometries homotopic to the identity map in the case of theT eichmüller space. Results about the homotopy groups of these spaces were also mentioned together with some applications to the study of smooth bundles with negatively curved fibers diffeomorphic to M and having a fixed base space B. Two such bundles are said to be equivalent if there is such a bundle with base $B \times [0,1]$ whose restriction to $B \times 0$ and $B \times 1$ are fiberwise isometric to the respective given bundles. It is natural is ask whether the “forget structure” map F to the underlying smooth M-bundles is (1) non-trivial, (2) onto, or (3) one-to-one. In the case the base space B is a sphere, Theorem 1 together with our results on the Teichmüller space of M show that F is frequently non-trival, but usually not onto, and frequently not one-to-one. And our results lead us to conjecture that such a bundle must be topologically trivial when B is simply connected.

Joel Hass, UC-Davis, *Harmonic maps and the Stabilization of Heegaard Splittings*

Area minimizing surfaces have proved to be a powerful tool in the study of 3-dimensional manifolds. Harmonic maps of surfaces to 3-manifolds have not been as widely applied, due to several limitations. A homotopy class of surfaces gives rise to a large space of harmonic maps, one for each conformal structure on its domain. These maps do not minimize their self-intersections, and may fail to be be immersed. Harmonic maps are somewhat better behaved, however, when the 3-manifold is negatively curved. In this setting smooth families of surfaces give rise to smooth families of harmonic maps, a property that cannot be expected from minimal surfaces, and that can be used to explore topological properties of the manifold. In this paper we study Heegaard splittings of 3-manifolds via families of harmonic surfaces.

A *genus g Heegaard splitting* of a 3-manifold M is a decomposition of M into two genus g handlebodies with a common boundary. It is described by an ordered triple (H_1, H_2, M) where each of H_1, H_2 is a handlebody in M and the two handlebodies intersect along their common boundary S, called a *Heegaard surface*. Two Heegaard splittings (H_1, H_2, M) and (H_1', H_2', M) are equivalent if an ambient isotopy of M carries (H_1, H_2) to (H_1', H_2'). Every 3-manifold has a Heegaard splitting and Heegaard splittings form one of the basic structures used to analyze and understand 3-manifolds.

Corresponding to the Heegaard splitting is a family of surfaces that sweep out the manifold, starting with a core of one handlebody and ending at a core of the second. This family is geometric controlled by deforming it to a family of harmonic maps. When the manifold is negatively curved, harmonic maps of genus g surfaces have uniformly bounded area. In the manifolds we construct, the geometry forces small area surfaces to line up with small area cross sections of the manifold. As a result we obtain obstructions to the equivalence of distinct Heegaard splittings.

A *stabilization* of a genus g Heegaard surface is a surface of genus $g + 1$ obtained by adding a 1-handle whose core is parallel to the surface. Such a surface splits the manifold into two genus $g + 1$ handlebodies, and thus gives a new Heegaard splitting. Two genus g Heegaard splittings are *k-stably equivalent* if they become equivalent after k stabilizations. Any two Heegaard splittings become equivalent after a sequence of stabilizations. An upper bound on the number of stabilizations needed to make two splittings equivalent is known in some cases. If G_p and G_q are splittings of genus p and q with $p \leq q$, and M is non-Haken, then Rubinstein and Scharlemann obtained an upper bound of $5p+8q-9$ for the genus of a common stabilization. But all previously known examples of manifolds with distinct splittings become equivalent after a single stabilization of the larger genus Heegaard surface. It has been conjectured that a single stabilization always suffices (See Kirby Problem List, Problem 3.89). This is sometimes called the *stabilization conjecture*. We show that this conjecture does not hold. There are pairs of genus g splittings of a manifold that require no fewer than g stabilizations to become equivalent.

Theorem. *For each $g > 1$ there is a 3-manifold M_g with two genus Heegaard g splittings that require g stabilizations to become equivalent.*

Chikako Mese, Johns Hopkins, *Harmonic Maps in Singular Spaces*

This report is joint work by Georgios Daskalopoulos and Chikako Mese.

The seminal work of M. Gromov and R. Schoen extends the study of harmonic maps between smooth manifolds to the case when the target is a Riemannian simplicial complex of non-positive curvature. The theory of harmonic maps into singular spaces was expanded substantially by the work of N. Korevaar and R. Schoen where they consider targets that are arbitrary metric spaces of non-positive curvature. (Such spaces are called NPC or CAT(0) if they are simply connected.) One important motivation for considering singular spaces in the theory of harmonic maps is in studying group representations. The main application of the Gromov-Schoen theory is to establish a certain case of non-Archemedean superrigidity complementing Corlette's Archemedean superrigidity for lattices in groups of real rank 1.

The next step in the generalization of the harmonic map theory is to replace smooth domains by singular ones. This problem is also motivated by superrigidity, in this case when the domain group is non-Archemedean. The consideration of a Riemannian simplicial complex as the domain space for harmonic maps seems to have been initiated by J-Y. Chen. Subsequently, this theory was further elaborated by J. Eells and B. Fuglede. In particular, they show Hölder continuity for harmonic maps under an appropriate smoothness asumption for the metric on each simplex.

Recall that the main idea of Gromov-Schoen is also to show that harmonic maps are regular enough so that Bochner methods could be used in the setting of singular targets. In particular, the fundamental regularity result of Gromov-Schoen and of Korevaar-Schoen is that harmonic maps from a smooth Riemannian domain into a NPC target are locally Lipschitz continuous. This statement no longer holds when we replace the domain by a polyhedral space. On the other hand, we have found that modulus of continuity better than Hölder is crucial in applications. This necessitates stronger regularity results than Hölder.

In our regularity theorems, we prove that harmonic maps from an admissible n-dimensional Riemannian complex into a NPC space is Lipschitz continuous away from the $(n-2)$-skeleton. Furthermore, near a point on a k-skeleton, we give explicit dependence of the Hölder exponent of a harmonic map on the combinatorial and geometric information of the link of the k-dimensional skeleton. Using this, we provide a sufficient criterion for a harmonic map to be Lipschitz continuous.

The development of the harmonic map theory from a Riemannian complex is important in the study of non-Archedemean lattices. We establish certain fixed point and rigidity theorems of harmonic maps Riemannian complexes. The key issue in the techniques we introduce is to prove regularity theorems strong enough to be able to apply differential geometric methods.

Hossein Namazi, University of Texas at Austin, *Splittings, Hyperbolic Geometry and Models*

This is joint work with J. Brock, Y. Minsky, and J. Souto.

The question of classifying the closed 3-manifolds is possibly the most important question which has motivated the research in the field of the low dimensional topol- ogy. This question saw a major turning point with Thurston's fundamental work in connecting the geometry and topology of 3-manifolds. In particular in his Geometrization Conjecture, Thurston conjectured that a closed 3-manifold can be decomposed (topologically) into pieces which admit one of eight possible geometric structures. He went further and not only proved this for a large class of 3-manifolds but also showed that "most" of these pieces cannot admit any of these geometric structure other than the hyperbolic structure. In particular to solve the conjecture it would be enough to prove these manifolds admit a hyperbolic structure. This remained the most important conjecture in the field until Perelman's recent claim in proving the conjecture using analytic methods.

In our work, we try to emphasize that Perelman's approach and our understanding of his proof lacks an effective construction of the hyperbolic metric. This is one reason why there are still many questions and conjectures regarding the topology of the 3-manifolds which remain unanswered even after assuming the Geometrization. In our work, we try to follow some of Thurston's original ideas and many new developments in the field to obtain a constructive approach to find and describe the hyperbolic metric. We explain how this approach provides answers to questions which remained unanswered even after knowing the Geometrization conjecture is true.

One of the most common ways of constructing 3-manifolds (topologically) is to start from a number of compact 3-manifolds (with boundary) and gluing them along their boundary components. Then by changing the gluing map, one can produce infinite families of such manifolds. A natural question is to ask how the combinatorics of the gluing maps affect the geometry of the final 3-manifold. We introduce a type of combinatorial information on the boundary components of these three manifolds which we call a *decoration* and we say a 3-manifold is *decorated* if every boundary component is equipped with one such decoration. Fixing a finite family of decorated compact 3-manifolds and a constant $R > 0$, we introduce a special type of *gluing called gluing with R-bounded combinatorics*. The introduction of such gluings is motivated by work of Masur-Minsky on the *complex of curves* of surfaces and also by Minsky's work in relating these to the geometry of ends of open hyperbolic 3-manifolds. our main theorem is the following:

Theorem 1. *Given a finite family $\mathcal{M}$ of decorated manifolds and $R > 0$ if a manifold X is obtained from copies of elements of $\mathcal{M}$ with "sufficiently complicated" gluings with R-bounded combinatorics, then if X*

is also hyperbolic we can describe the geometry of X with its hyperbolic metric using a bi-Lipschitz model.

In order to prove this theorem we use some convergence theorems which are gener- alizations of Thurston's double limit theorem. These however owe more to Morgan- Shalen's approach in proving Thurston's theorem. We describe a generalization of Morgan-Shalen work and then after applying other results by Skora, Namazi-Souto and Kleineidam-Souto, we prove our convergence result. We use this convergence results for hyperbolic structures on the interiors of decorated manifolds in $\mathcal{M}$. It follows from those that with a suitable choice of such structures, one can glue these and obtain a negatively curved metric on X whose sectional curvatures are pinched between $-1-\epsilon$ and $-1+\epsilon$ for $\epsilon > 0$ small. Besides the geometry of these metrics can be described using a combinatorial models.

The negatively curved metrics allow us to use the convergence results for the rep- resenations that are induced from a hyperbolic structure on manifolds obtained by R-bounded combinatorial gluings of elements of M. Then using the convergence results we show that the hyperbolic metric is also bi-Lipschitz to the constructed combinatorial model. This finishes our main theorem.

Our main application of the main theorem is for the Heegaard splittings. Given $g > 1$, we use the theorem to introduce a class of splittings which we call *splittings with generalized R-bounded combinatorics*. Then we show that

Theorem 2. *Given R there exists $\epsilon > 0$ such that if a hyperbolic 3-manifoldd admits a genus g splitting with R-bounded combinatorics, then its injectivity radii at all points are at least ϵ. Vice versa, given $\epsilon > 0$ there exists R such that if all the injectivity radii at all points of a hyperbolic 3-manifold are at least ϵ and the manifold has a genus g splitting then the splitting must have generalized R-bounded combinatorics. Even more if a hyperbolic 3-manifold satisfies any of the above, it is bi-Lipschitz to the described combinatorial model.*

Yi Ni, A.I.M.& M.I.T., *Dehn Surgeries that Reduce the Thurston Norm of a Fibered Manifold*

Dehn surgery is an important method of constructing 3-manifolds: basically all 3manifolds can be constructed via it. A general question about Dehn surgery is: how do the invariants of 3-manifolds behave under Dehn surgeries? In this talk, we will consider the behavior of Thurston norm under Dehn surgeries.

Definition. Let S be a compact oriented surface with connected components $S_1,\ldots,S_n$. We define

$$x(S) = \sum_i \max\{0, -\chi(S_i)\}.$$

Let M be a compact oriented 3manifold, let $h \in H_2(M, \partial M)$. The *Thurston norm* $x(h)$ of h is defined to be the minimal value of $x(S)$, where S runs over all the properly embedded surfaces in M with $[S] = h$.

A very important result about Thurston norm is due to Thurston: suppose S is a compact leaf of a taut foliation of a 3manifold, then S is Thurston norm minimizing in its homology class. Later works of Gabai made this result practical by constructing taut foliations with prescribed compact leaves.

When Y ($\neq S^2 \times S^1$) is a surface bundle over S^1, the fibration is a taut foliation, and every fiber is a compact leaf of the foliation. So Thurstons theorem says that the fiber of the surface bundle is Thurston norm minimizing in Y.

Here is a simple fact:

Fact. Suppose $F \subset Y$ is a connected Thurston norm minimizing surface, $x(F) > 0$. $K \subset Y$ is a knot on F, and λ is the slope on K specified by F. Let X be the manifold obtained by doing λsurgery on F, then F (viewed as a surface in X) is compressible in X. Let x_X, x_Y denote the Thurston norm in X and Y, respectively, then

$$x_X(X([F]) < x_Y([F]).$$

Our main theorem is a converse to the above fact in the case when Y is fibered.

Theorem. *Suppose Y is a surface bundle over the circle, and F is a fiber. Let $K \subset Y$ be a knot such that $[K] \cdot [F] = 0$, and let α be a slope on K. Let X be the manifold obtained by αsurgery on K. Since $[K] \cdot [F] = 0$, $[F]$ can also be viewed as a homology class in $H_2(X; \partial X)$.*

Let x_X, x_Y denote the Thurston norm in X and Y, respectively. If

$$x_X([F]) < x_Y([F]),$$

then there is an ambient isotopy of Y which takes K to a curve in F. Moreover, the slope α coincides with the frame on K which is specified by the surface F.

The proof of the above theorem uses Gabais sutured manifold theory, as well as an argument introduced by Ghiggini in contact topology.

Stefano Vidussi, UC Riverside, *Twisted Alexander Polynomials and Fibered 3-Manifolds*

Symplectic 4-manifolds are perhaps the best understood class of symplectic manifolds, but in spite of this fact we still do not have, even at conjectural level, a classification scheme, and are unable to decide in general whether a given 4–manifold admits or not a symplectic structure. A particular class of manifolds where, conceivably, the problems above can be tackled is the class of 4-manifolds that admit a circle action. We focus on a particular case, namely product manifolds of the form $S^1 \times N$, where N is a closed 3-manifold. About 30 years ago William Thurston proved that if N admits a fibration over S^1, then $S^1 \times N$ admits a symplectic structure, and it has long been conjectured that this condition is not only sufficient but also necessary. Seiberg-Witten theory, especially through the work of Cliff Taubes for the symplectic case, provides a tool to decide this question. In collaboration with Stefan Friedl, we have used the information arising from the Seiberg-Witten invariants of $S^1 \times N$ (and all its finite covers) to show that a family of invariants of N, the twisted Alexander polynomials, satisfy rather strict constraints, that where previously known to holds for fibered 3-manifolds. These constraints are the analog of a well-known condition on the Alexander polynomial of a fibered knot, namely that the polynomial is monic and its degree equals the genus of the knot. Led by this result, and numerical evidence, we have conjectured that these constraints completely characterize fibered 3-manifolds.

In the talk I present a proof of this conjecture, obtained in collaboration with Friedl. The strategy can be summarized as follows. First, it is known that the proof can be reduced to the case where N is an irreducible manifold. Denote by Σ a connected minimal genus surface that represent a primitive homology class in $H_2(N)$ for which the constraints are satisfied. Denote by M the exterior of Σ, i.e. $M = N \setminus \nu\Sigma$. Then M is a manifold with boundary given by two copies of Σ. It is well known that, with either inclusion, there is an injective map $\pi_1(\Sigma) \to \pi_1(M)$. We show that the aforementioned constraints imply that this map induces an isomorphism of prosolvable completions. Then, under the assumption that the fundamental group of N is residually finite solvable, we can show that the isomorphism of prosolvable completions actually implies that $M = \Sigma \times I$ (i.e. that N fibers with fiber Σ) by implementing the criterion for virtual fibrations recently determined by Ian Agol. In general, the fundamental group of a 3–manifold is not residually finite solvable, but by going to a suitable finite cover of N we can obtain a property that is close enough to allow one to apply the result above.