

Texas Geometry and Topology Conference

This is a report on the presentations at the 42nd meeting of the Texas Geometry and Topology Conference at Rice University, November 6-8, 2009. This conference was partially supported by National Science Foundation Grant DMS-0904481 and Rice University. Speakers reported on recent research. For this report, speakers have provided synopses of their talks together with broader discussions of the significance and context of their results.

Meeting 42. Rice University, November 6-8, 2009

Danny Calegari, Caltech, *Faces of the Scl Norm Ball*

It often happens that the solution of an extremal topological problem leads to geometric rigidity. I will show how a simple topological problem — when does an immersed curve on a surface bound an immersed subsurface? — is unexpectedly related to linear programming in Banach spaces, and rigidity for symplectic representations of surface groups.

Rafael de la Llave, University of Texas, Austin, *Geometric Properties of the Scattering Map to Normally Hyperbolic Manifolds and Applications to Instability*

Given a normally hyperbolic invariant manifold, whose stable and unstable manifolds intersect transversally, we can consider the map that to an orbit asymptotic in the past associates the orbit asymptotic in the future (the scattering map).

When the dynamics is symplectic, this map is symplectic and there are very natural perturbative formulas.

When there is a normally hyperbolic manifold with such intersections, by combining homoclinic excursions and staying close to the manifold, we can construct orbits that move very long distances over time even if the map is close to integrable. This is a version of Arnold diffusion.

The work presented is joint work with A. Delshams and T. M.-Seara.

Anton Gorodetski, UC Irvine, *On Quantitative Characteristics of Cantor Sets and Their Applications*

We will consider the class of so called “dynamically defined” Cantor sets. All of these sets have zero Lebesgue measure, so in order to study them one has to use other quantitative characteristics, such as Hausdorff dimension, box counting dimension, thickness, and denseness. We will discuss relations between these quantities, as well as some recent applications to conservative dynamics, celestial mechanics, and mathematical physics.

Rostislav Grigorchuk, Texas A&M University, *Ergodicity of Boundary Actions, Hopf Decomposition and Nielsen-Schreier Method*

This is a joint work with V. Kaimanovich and T. Nagnibeda.

I will speak about ergodic properties (ergodicity and conservativity) of the action of a subgroup of a free group on its boundary with respect to the uniform measure. The Hopf decomposition of the boundary action will be described in terms of Nielsen-Schreier theory (one of the main tools in combinatorial group theory). Growth and cogrowth will be used to characterize conservativity and dissipativity. Amenability and Liouville property will be mentioned to illustrate some cases.

Mike Hochman, Princeton University, *Local Entropy Averages and Projection of Fractal Measures*

If X is a compact set in the plane then, by a classical theorem of Marstrand, almost every projection onto a line maps X to a set of the maximal possible dimension; i.e., the smaller of $\dim(X)$ and 1. An old conjecture of Furstenberg's predicts that, if $X = A \times B$, and A, B are, respectively, subsets of the unit interval invariant under times-2 (mod 1) and times-3 (mod 1), then the image of X under every projection should behave this way, the only exceptions being the coordinate projections. I will describe recent work with Pablo Shmerkin in which we resolve this conjecture and derive some related results.

Mikhail Lyubich, SUNY Stony Brook, *Two-Dimensional Rational Dynamics and Lee-Yang Zeros*

In a classical work of 1950's, Lee and Yang proved that zeros of certain polynomials (related to models of magnetism) lie on the unit circle. Distribution of these zeros is physically important as it controls phase transitions in the model. We study this distribution for a special "Diamond Hierarchical Lattice." In this case, it can be described in terms of the dynamics of an explicit rational map in two variables. We prove partial hyperbolicity of this map on an invariant cylinder, and derive from it that the Lee-Yang zeros are organized asymptotically in a transverse measure for its central foliation. From the global complex point of view, the zero distributions get interpreted as slices of the Green (1,1)-current on the projective space. Some of these geometric results are obtained by means of "enumerative dynamics", by counting intersections of certain algebraic curves.

It is a joint work with Pavel Bleher and Roland Roeder.

Karin Melnick, University of Maryland, *Normal Forms for Conformal Vector Fields*

Isometries of Riemannian or pseudo-Riemannian manifolds are linearizable in the neighborhood of a fixed point via the exponential map. Conformal transformations, on the other hand, are not linearizable in general. I will present recent work with C. Frances toward normal forms for conformal vector fields on pseudo-Riemannian manifolds for which the flow has bounded differential at a singularity. In particular, when the metric is real-analytic, we show such a flow either is linearizable, or the manifold is conformally flat, and the flow is locally conjugate to a conformal flow on the corresponding flat Minkowski space.