

Second Writing Assignment, Math220

- (1) Let $n \in \mathbb{N}$.
 - (a) For all $a \in \mathbb{Z}$, $a \equiv a \pmod{n}$.
 - (b) For all $a, b \in \mathbb{Z}$, if $a \equiv b \pmod{n}$ then $b \equiv a \pmod{n}$.
 - (c) For all $a, b, c \in \mathbb{Z}$ if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.
- (2) Let $n \in \mathbb{N}$. For $a, b \in \mathbb{Z}$, show that if $a \equiv b \pmod{n}$ then $a^2 \equiv b^2 \pmod{n}$.
- (3) For $d \in \mathbb{N}$ we define
$$A_d = \{n \in \mathbb{Z} \mid d \mid n\} \text{ and } B_d = \{n \in \mathbb{Z} \mid d \mid n^2\}.$$
 - (a) Find an example of $d \in \mathbb{N}$ for which $A_d = B_d$, and find an example for which $A_d \neq B_d$.
 - (b) State a condition on d which is equivalent to the statement that $A_d = B_d$ and prove it.