Problems in Functional Analysis (Math655)
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Please attempt 6 ouf the following 12 problems.
From Book:
Page 26/1.31, page 29/1.38, page 53/2.8, page 55/2.16, page 57/2/38.

(1) Show the equivalences for being an extreme point stated in the Proposition before the Krein-Milman Theorem.
(2) Show that for two finite dimensional Banach spaces X and Y:
$d_{BM}(X,Y) = 1$ if and only if there is an isometry between X and Y.
(3) $(*)$ Show that there are two Banach spaces X and Y for which $d_{BM}(X,Y) = 1$ but there is no isometry between X and Y.
(4) Let X be an infinite dimensional Banach space.
   (a) Show that every weak null sequence in X is bounded.
   (b) Show that there is an unbounded net in X which converges weakly to 0.
   (c) Show that the weak topology is not metrizable on all of X.
(5) Find the extreme points of $B_{C([0,1])}$. Hint: the cases $\mathbb{K} = \mathbb{R}$ and $\mathbb{K} = \mathbb{C}$ differ.
(6) If $\mathbb{K} = \mathbb{C}$ prove that $B_{C([0,1])} = \overline{\mathrm{co}(\mathrm{ext}(B_{C([0,1])}))}$.
(7) (*) If $\mathbb{K} = \mathbb{C}$ prove that $B_{C(D)} = \overline{\mathrm{co}(\mathrm{ext}(B_{C(D)}))}$, where D is the complex disc, i.e. $D = \{z \in \mathbb{C} : |z| \leq 1\}$.

The previous problem is a special case of the following result (don't worry if you don't know the terminology, you will learn it during the net semester):

Theorem 0.1 (Russo Dye)**.** *Let $\mathcal{A}$ be a unital C^*-algebra, and denote the sets of its unitaries by $\mathcal{U}(\mathcal{A})$ then*

$$B_{\mathcal{A}} = \overline{\mathrm{ext}(\mathrm{co}(\mathcal{U}(\mathcal{A})))}.$$