Please attempt 6 out of the following 12 problems.

From Book:
Page 26/1.31, page 29/1.38, page 53/2.8, page 55/2.16, page 57/2/38.

1. Show the equivalences for being an extreme point stated in the Proposition before the Krein-Milman Theorem.

2. Show that for two finite dimensional Banach spaces $X$ and $Y$:
   
   $$d_{BM}(X, Y) = 1$$
   
   if and only if there is an isometry between $X$ and $Y$.

3. (*) Show that there are two Banach spaces $X$ and $Y$ for which $d_{BM}(X, Y) = 1$ but there is no isometry between $X$ and $Y$.

4. Let $X$ be an infinite dimensional Banach space.
   
   (a) Show that every weak null sequence in $X$ is bounded.
   
   (b) Show that there is an unbounded net in $X$ which converges weakly to $0$.
   
   (c) Show that the weak topology is not metrizable on all of $X$.

5. Find the extreme points of $B_{C([0,1])}$. Hint: the cases $K = \mathbb{R}$ and $K = \mathbb{C}$ differ.

6. If $K = \mathbb{C}$ prove that $B_{C([0,1])} = \text{co}(\text{ext}(B_{C([0,1])}))$.

7. (*) If $K = \mathbb{C}$ prove that $B_{C(D)} = \text{co}(\text{ext}(B_{C(D)}))$, where $D$ is the complex disc, i.e. $D = \{z \in \mathbb{C} : |z| \leq 1\}$.

The previous problem is a special case of the following result (don’t worry if you don’t know the terminology, you will learn it during the net semester):

**Theorem 0.1** (Russo Dye). Let $A$ be a unital $C^*$-algebra, and denote the sets of its unitaries by $U(A)$ then

$$B_A = \text{ext}(\text{co}(U(A)))$$.