Problems in Functional Analysis (Math655)
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Please attempt 6 ouf the following 12 problems.
From Book
page 86 ff: 3.63, 3.70, 3.83, 3.90
page 130ff: 4.4, 4.19

(1) Show that the unit ball in ℓ_∞^* is not sequentially compact in w^*, i.e. not every sequence has a converging subsequence.
(2) Prove the *non linear Hahn Banach Theorem*:
X is a metric space $Y \subset X$ and $f : Y \to \mathbb{R}$ Lipschitz cntinuous. Then f cn be extended to a Lipschitz function $F : X \to Y$ having the same Lipschitz constant as f.
(3) (*) Prove above non linear Hahn Banach Theorem without using the axiom of choice.
(4) Let X be a Banach space, we call a set $A \subset X$ *limited in* X if every sequence (x_n^*) in X^* converging w^* to 0 converges uniformly to 0 on A.
Prove that relatively compact sets are limited and that if X is separable every limited set is relatively compact.
(5) (*) Find a Banach space X which admits limited sets which are not relatively compact.
(6) Let X be a Banach space. Show that $(X^*, w^*)^* = (X, w)$.