Problems in Real Analysis II (Math608)  Due: 4/21/10

**Problem 1.** Every Hilbert space $H$ has an orthonormal basis. If $H$ is separable every orthonormal basis is countable.

**Problem 2.** For $n \in \mathbb{Z}$, define
\[
  f_n : [-\pi, \pi] \to \mathbb{C}, \quad x \mapsto \frac{1}{2\pi} e^{int} = \frac{1}{\sqrt{2\pi}} (\cos(nt) + i\sin(nt)).
\]
Show that $(f_n)$ is an orthonormal basis of $L_2[-\pi, \pi]$.

**Problem 3.** Problem 66/page 178

**Problem 5.** Every closed convex set $K \neq \emptyset$ in a Hilbert space has a unique element of minimal norm.  
**Hint:** adapt proof of Theorem 5.24.

**Problem 6.** Problem 56/page 177.

**Problem 7.** Problem 58/page 177.