

Problems in Real Analysis II (Math608)

Due: 4/21/10

Problem 1. Every Hilbert space H has an orthonormal basis. If H is separable every orthonormal basis is countable.

Problem 2. For $n \in \mathbb{Z}$, define

$$f_n : [-\pi, \pi] \rightarrow \mathbb{C}, \quad x \mapsto \frac{1}{2\pi} e^{int} = \frac{1}{\sqrt{2\pi}} (\cos(nt) + i \sin(nt)).$$

Show that (f_n) is an orthonormal basis of $L_2[-\pi, \pi]$.

Problem 3. Problem 66/page 178

Problem 5. Every closed convex set $K \neq \emptyset$ in a Hilbert space has a unique element of minimal norm.

Hint: adapt proof of Theorem 5.24.

Problem 6. Problem 56/page 177.

Problem 7. Problem 58/page 177.