Problems in Real Analysis II (Math608) **Due: 4/28/10**

Problem 1. Show that for a LCH space X

$$\overline{C_c(X)} = \left\{ f : X \to \mathbb{F}, \quad \begin{matrix} \text{continuous and bounded, and} \\ \{x \in X : |f(x)| \geq \varepsilon\} \text{ is compact for all } \varepsilon > 0 \end{matrix} \right\}.$$

where the closure is taken in $C_b(X)$ with respect to $\|\cdot\|_u$.

Problem 2. Show that for a bounded sequence $(f_n) \subset C[0,1]$,

$$f_n \to_{n\to\infty} 0 \text{ weakly} \iff \forall t \in [0,1] \quad f_n(t) \to_{n\to\infty} 0.$$

Problem 3. 7/page 220