HOMEWORK #1

Exercise 8 Pg 13:

(a) A real-valued function f is *not bounded* on $[a,b]$ if for every positive real number M, there exists $x \in [a,b]$ such that $|f(x)| > M$.

(b) The function $f(x) = x$ is bounded on $[0,1]$.

(c) The function $f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is unbounded on $[0,1]$

(d) The reworded definition does not mean the same as the one given in Example 14. The original definition requires that the real number M not depend on x. In other words, for the real number M, the inequality $|f(x)| \leq M$ must hold for all $x \in [a,b]$. By rewording the definition so that the phrase "for all $x \in [a,b]$" precedes the phrase "there exists a positive real number M" the M may change depending on the value of x. In fact if we just let $M = |f(x)|$, every function would satisfy this "definition" of boundedness.

Exercise 10 pg 14:

(a) A real-valued function $f(x)$ is said to be ~~deaer~~ decreasing on the closed interval $[a,b]$ if for all $x_1, x_2 \in [a,b]$ if $x_1 < x_2$, then $f(x_1) > f(x_2)$.

(b) A real-valued function f is not decreasing on $[a,b]$ if there exists $x_1, x_2 \in [a,b]$ such that $x_1 < x_2$ and $f(x_1) \leq f(x_2)$.

(c) The function $f(x) = 1 - x$ is decreasing on $[0,1]$.

Exercise 10 cont.

(d) The function $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ ~~in~~ is neither increasing nor decreasing on $[0,1]$.

Exercise 8 Pg 26:

"n and m are both even or both odd"

Exercise 16 Pg 26:

(a) If P is true, then eithe $P \wedge Q$ or $P \wedge \neg Q$ is true depending on whether or not Q is true or false. Hence if P is true, the statement form $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$ is true. On the other hand, if P is false, then either $\neg P \wedge Q$ or $\neg P \wedge \neg Q$ is true again depending on the truth or falsity of Q. Thus if P is false, $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$ is true. It follows that $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

(b) If P is true, then either $\neg P \vee Q$ or $\neg P \vee \neg Q$ is false depending on the truth or falsity of Q. Thus $(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (\neg P \vee \neg Q)$ is false. If P is false then either $P \vee Q$ or $P \vee \neg Q$ is false. So in this case also, $(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (\neg P \vee \neg Q)$ is false. Therefore, $(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (\neg P \wedge \neg Q)$ is a contradiction.